1

Sets

A bookcase is among the best illustrations of sets. You maintain a set order for your book collection, whether it be alphabetical, genre-based, or based on personal preferences. As a result, collections of associated books are kept apart from one another.

Topic Notes

- *Basic Concepts of Sets*

BASIC CONCEPTS OF SETS 1

TOPIC 1

SETS AND THEIR REPRESENTATION

In our day-to-day life, we come across collections of objects of a particular type, such as days in a week, months in a year, playing cards in a pack, vowels in the English alphabet, etc. In mathematics also, we come across collections, for example, collections of natural numbers, prime numbers, etc.

There are a few collections given below:

A collection of whole numbers less than 10.

A collection of all the rivers in Uttar Pradesh.

A collection of prime factors of 64.

A collection of prime ministers of India.

We can see these are the collections of well-defined objects in the sense that we can definitely decide whether a given particular object belongs to a given collection or not. However, it may not always be the case. For instance, the collection of the five most renowned mathematicians of the world, is not well-defined, because the criterion for determining a mathematician as most renowned may vary from person to person. Thus, it is not a well-defined collection, and hence not a set, as defined in mathematics. Any type of object can be collected into a set, but the set theory is applied more often to objects that are relevant to Mathematics.

Hence, we can say that a set is a well-defined collection of objects.

Any type of object can be collected into a set, but the set theory is applied more often to objects that are relevant to Mathematics.

Illustration: The collection of the first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

Illustration: The solution of the equation: $x^2 - 5x + 6 = 0$, *viz*, 2 and 3 is a set of solutions.

Illustration: The collection of all states including all union territories in the Indian Union is a set.

Illustration: The collection of cricketers in the world who were out for 99 runs in a test match is a set.

Some more examples of sets used particularly in mathematics, viz.

N : Set of all-natural numbers

Z : Set of all integers

Z^+ : Set of all positive integers

Q : Set of all rational numbers

Q^+ : Set of all the positive rational numbers

R^+: Set of all positive real numbers

C : Set of all complex numbers.

The following points may be noted:

(1) Objects, elements and members of a set are synonymous terms.

(2) Sets are usually denoted by capital letters A, B, C, X, Y, Z, etc.

(3) The elements of a set are represented by small letters *a*, *b*, *c*, *x*, *y*, *z*, etc.

(4) If a is an element of set A, we say that "a belongs to A" and we write '$a \in A$'. Here '$\in$' (Epsilon) is a Greek symbol that is used to denote the phrase 'belongs to'. If *b* is not an element of set A. We say that '*b* does not belong to A' and we write '$b \notin A$'.

Caution

➥ *Students should remember that, in sets, $\in$ means 'belongs to' and '$\notin$' means "does not belong to".*

Methods of Representation of Sets

A set can be represented in two ways:

Roster Form or Tabular Form

In roster or tabular form all the elements of a set are listed, separated by commas and are enclosed within curly brackets { }.

Illustration: Let A be the set of all letters in the word 'MATH'. Then, set A can be written in roster form as

$$A = \{M, A, T, H, S\}.$$

Illustration: Let A be the set of all vowels in the English alphabet, then $A = \{a, e, i, o, u\}$

Important

➥ *In roster form, the order in which the elements are listed is immaterial.*

Set-Builder Form

In set-builder form, all the elements of a set possess a single common property that is not possessed by any element outside the set. In this form the descriptions of the elements of the set is made by using a symbol *x* which is followed by a colon ':' or a vertical bar '|' (read as such that) and then enclosing the whole description within brackets { }.

Illustration: If *B* is the set of all even integers then *B* can be written in set-builder form as

$$B = \{x : x = 2x, x \in Z\}$$

Or

$$B = \{x \mid x = 2x, x \in Z\}$$

Illustration: If A is a set of squares of natural numbers then A can be written in set-builder form as

$A = \{x: x \text{ is the square of natural number}\}$

Or $A = \{x: x = n^2, \text{ where } n \in N\}$

Example 1.1: **Write the following sets in roster form:**

(A) $A = \{x : x$ **is an integer and** $-3 \le x < 7\}$.

(B) $B = \{x : x$ **is a natural number less than 6**$\}$.

(C) $C = \{x : x$ **is a two-digit natural number such that the sum of its digits is 8**$\}$.

(D) $D = \{x : x$ **is a prime number which is a divisor of 60**$\}$.

(E) E **= The set of all letters in the word TRIGONOMETRY.**

(F) F **= The set of all letters in the word BETTER.** **[NCERT]**

Ans. (A) $A = \{x : x$ is an integer and $-3 \le x < 7\}$.

The elements of set A are $-3, -2, -1, 0, 1, 2, 3, 4, 5$ and 6 only.

Therefore, the roster form of set

$A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(B) $B = \{x : x$ is a natural number less than 6$\}$.

The elements of set B are 1, 2, 3, 4 and 5 only.

Therefore, the roster form of set

$B = \{1, 2, 3, 4, 5\}$

(C) $C = \{x : x$ is a two-digit natural number such that the sum of its digits is 8$\}$.

The elements of set C are 17, 26, 35, 44, 53, 62 and 71 only.

Therefore, the roster form of set

$C = \{17, 26, 35, 44, 53, 62, 71\}$

(D) $D = \{x : x$ is a prime number which is divisor of 60$\}$.

Prime number which are divisor of 60 are:

$60 \div 2 = 30$, $60 \div 3 = 20$, and $60 \div 5 = 12$

Therefore, the elements of the set D are 2, 3, and 5.

Hence, the roster form of set $D = \{2, 3, 5\}$

(E) E = The set of all letters in the word TRIGONOMETRY

There are 12 letters in the word TRIGONOMETRY.

Out of which letters T, R, and O are repeated.

Therefore, the roster form of set

$E = \{T, R, I, G, O, N, M, E, Y\}$

(F) F = The set of all letters in the word BETTER.

There are 6 letters in the word BETTER, out of which letters E and T are repeated.

Therefore, the roster form of set $F = \{B, E, T, R\}$

Example 1.2: **Write the following sets in the set-builder form:**

(A) $\{3, 6, 9, 12\}$ **(B)** $\{2, 4, 8, 16, 32\}$

(C) $\{5, 25, 125, 625\}$ **(D)** $\{2, 4, 6, ...\}$

(E) $\{1, 4, 9, ..., 100\}$ **[NCERT]**

Ans. (A) Let $A = \{3, 6, 9, 12\}$

All elements of the set A are natural numbers that are multiples of 3 having value less than or equal to 12.

$\therefore$ The set-builder form of set $A = \{x : x = 3n, n \in N \text{ and } 1 \le n \le 12\}$.

(B) Let $B = \{2, 4, 8, 16, 32\}$

All elements of the set B are natural numbers that are powers of 2.

$\therefore$ The set-builder form of set $B = \{x : x = 2^n, n \in N \text{ and } 1 \le n \le 5\}$.

(C) Let $C = \{5, 25, 125, 625\}$

All the elements of the set C are natural numbers that are powers of 5.

$\therefore$ The set-builder form of set $C = \{x : x = 5^n, n \in N \text{ and } 1 \le n \le 4\}$.

(D) Let $D = \{2, 4, 6, ...\}$

All the elements of the set D are even natural numbers.

$\therefore$ The set-builder form of set $D = \{x : x$ is an even natural number$\}$.

(E) Let $E = \{1, 4, 9, ..., 100\}$

All the elements of the set E are perfect square numbers.

$\therefore$ The set-builder form of set $E = \{x: x = n^2 \text{ and } 1 \le n \le 10\}$.

Example 1.3: **Let A = {1, 2, 3, 4, 5, 6}. Insert the appropriate symbol** $\in$ **or** $\notin$ **in the blank spaces:**

(A) $5 \ldots A$ **(B)** $8 \ldots A$

(C) $0 \ldots A$ **(D)** $4 \ldots A$

(E) $2 \ldots A$ **(F)** $10 \ldots A$

Ans. Given $A = \{1, 2, 3, 4, 5, 6\}$, Therefore,

(A) Here, 5 is in set A. Therefore, $5 \in A$

(B) Since, 8 is not in set A. Therefore, $8 \notin A$

(C) Since, 0 is not in set A. Therefore, $0 \notin A$

(D) Since, 4 is in set A. Therefore, $4 \in A$

(E) Since, 2 is in set A. Therefore, $2 \in A$

(F) Since, 10 is not in set A. Therefore, $10 \notin A$

TOPIC 2

TYPES OF SETS

Different types of sets are classified according to the number of elements they have. Let's now learn the types of sets in this topic.

Empty Set

A set that does not contain any element is called the empty set or the null set or the void set. It is denoted by the symbol ϕ (phi) or { }.

E.g. $A = \{x: x$ is a natural number less than $1\}$, there is no natural possibility that it is less than 1. Hence, set A is an empty or null set, i.e., $A = \phi$ or { }.

Example 1.4: Which of the following are examples of the null set?

(A) Set of odd natural numbers divisible by 2.

(B) Set of even prime numbers.

(C) $\{x : x$ is a natural numbers, $x < 5$ and $x > 7\}$

(D) $\{y : y$ is a point common to any two parallel lines$\}$ [NCERT]

Ans. (A) A set of odd natural numbers divisible by 2 is a null set because no odd number is divisible by 2.

(B) Let A be a set of even prime numbers.

Since, 2 is only even prime number, i.e., $A = \{2\}$. Hence, it is not an empty or null set.

(C) Let $A = \{x : x$ is a natural number, $x < 5$ and $x > 7\}$.

There is no natural number possible which is less than 5 and greater than 7. Hence, A is a null set, i.e., $A = \{ \}$.

(D) $\{y : y$ is a point common to any two parallel lines$\}$ is a null set because parallel lines do not intersect. Hence, they have no common point.

Singleton Set

A set that contains only one element is called a singleton set.

Illustration: Let $A = \{x : x + 5 = 0, x \in Z\}$ is a singleton set as set A contains only one integer -5.

Caution

➥ *Students should understand and remember that {0} is a singleton set not an empty set ({ } or ϕ).*

Finite and Infinite Sets

A set which is empty or consists of a definite number of elements, is called a finite set otherwise is called an infinite set.

Example 1.5: State which of the following sets are finite or infinite:

(A) $\{x : x \in N$ and $(x - 1)(x - 2) = 0\}$

(B) $\{x : x \in N$ and $2x - 1 = 0\}$

(C) $\{x : x \in N$ and x is prime$\}$

(D) $\{x : x \in N$ and x is odd$\}$

Ans. (A) Given set = {1, 2}. Hence, it is finite.

(B) Given set = ϕ. Hence, it is finite.

(C) The given set is the set of all prime numbers and since the set of prime numbers is infinite. Hence, the given set is infinite.

(D) Since, there are infinite numbers of odd numbers, hence, the given set is infinite.

Equal Sets

Two sets A and B are said to be equal if they have the same elements, and we write $A = B$, otherwise two sets are said to be unequal sets, i.e., $A \neq B$.

Equivalent Set

Two finite sets A and B are said to be equivalent, if they have the same number of elements, i.e., $n(A) = n(B)$ we can write $A \equiv B$ or $A \leftrightarrow B$.

Illustration: If $A = \{1, 2, 3, 4, 5\}$ and $B = \{a, e, i, o, u\}$

Since, $n(A) = n(B) = 5$, so A and B are equivalent sets.

Important

➥ *The number of distinct elements in a finite set is called the cardinal number of the set and it is denoted by n (A). E.g., if A = {1, 2, 3, 4, 5}, then n (A) = 5.*

Example 1.6: In the following, state whether $A = B$ or not:

(A) $A = \{2, 4, 6, 8, 10\}$, $B = \{x : x$ is positive even integer and $x \leq 10\}$.

(B) $A = \{x : x$ is a multiple of $10\}$; $B = \{10, 15, 20, 25, 30, \ldots\}$.

Ans. (A) $A = \{2, 4, 6, 8, 10\}$

$B = \{x : x$ is a positive even integer and $x \leq 10\}$.

Positive even integers less than equal to 10 = 2, 4, 6, 8, 10.

Hence, $B = \{2, 4, 6, 8, 10\}$

Elements of set A = 2, 4, 6, 8, 10

Elements of set B = 2, 4, 6, 8, 10

Since, all elements of set A are B are the same.

Hence, $A = B$

(B) $B = \{10, 15, 20, 25, 30 \ldots\}$

And $A = \{x : x$ is a multiple of $10\}$.

Multiples of 10 are = 10, 20, 30, 40, 50, . . .

Therefore, $A = \{10, 20, 30, 40, 50, \ldots\}$

Here, 15 and 25 are in set B but not in set A, i.e., $15, 25 \in B$ but $15, 25 \notin A$.

Hence, $A \neq B$

Example 1.7: From the sets given below, select equal sets:

$A = \{2, 4, 8, 12\}$, $B = \{1, 2, 3, 4,\}$, $C = \{4, 8, 12, 14\}$, $D = \{3, 1, 4, 2\}$, $E = \{-1, 1\}$, $F = \{0, a\}$, $G = \{1, -1\}$, $H = \{0, 1\}$. [NCERT]

Ans. $A = \{2, 4, 8, 12\}$, $n(A) = 4$

$B = \{1, 2, 3, 4\}$, $n(B) = 4$

$C = \{4, 8, 12, 14\}$, $n(C) = 4$

$D = \{3, 1, 4, 2\}$, $n(D) = 4$

$E = \{-1, 1\}$, $n(E) = 2$

$F = \{0, a\}$, $n(F) = 2$

$G = \{1, -1\}$, $n(G) = 2$

$H = \{0, 1\}$, $n(H) = 2$

Number of elements in A, B, C, D are the same, i.e., all have 4 elements. So they are comparable.

Now, we see that B and D have the same elements.

So, B and D are equal sets.

Similarly, E, F, G, H are comparable as they all have the same number of elements i.e. 2.

Clearly, E and G have the same elements.

So, E and G are equal sets.

TOPIC 3

SUBSETS AND UNIVERSAL SET

Subsets

Let A and B are two sets. If every element of set A is an element of set B, then we can say that 'A is called a subset of B' or 'A is contained in B', i.e., A set A is said to be a subset of a set B if every element of A is also an element of B and we denote it by writing $A \subset B$.

The symbol '$\subset$' is used to denote the phrase 'is a subset of' or 'is contained in'.

In notational form, we can write the following as:

$A \subset B$ if $a \in A \Rightarrow a \in B$.

The symbol '$\Rightarrow$' is used to denote the word 'implies that '.

So we read the above statement as "A is a subset of B, if a belongs to A implies that a also belongs to B".

If there exists at least one element of A which is not an element of B, then we say that, 'A is not a subset of B' or 'A is not contained in B' and we denote it by writing $A \not\subset B$.

We may note that for A to be a subset of B all that is needed is that every element of A is also in B. It is possible that every element of B may or may not be in A. If it so happens that every element of B is also in A, then we shall also have $B \subset A$. In this case, A and B are the same sets so that we have $A \subset B$ and $B \subset A \Leftrightarrow A = B$, where "$\Leftrightarrow$" is a symbol for two way implications, and is usually read as if and only if (briefly written as "iff").

Illustrations:

(1) The set Q of rational numbers is a subset of the set R of real numbers, and we write $Q \subset R$.

(2) If A is the set of all divisors of 56 and B is the set of all prime divisors of 56, then B is a subset of A and we write $B \subset A$.

(3) Let $A = \{1, 3, 5\}$ and $B = \{x : x$ is an odd natural number less than 6$\}$. Then $A \subset B$ and $B \subset A$ and hence $A = B$.

Let A and B be two sets. If $A \subset B$ and $A \neq B$, then A, is called a proper subset of B and B is called superset of A. For example, $A = \{1, 2, 3\}$ is a proper subset of $B = \{1, 2, 3, 4\}$. It is denoted by $A \subset B$.

A subset of a set A can be equal to set A but a proper subset of a set A can never be equal to set A. A proper subset of a set A is a subset of A that cannot be equal to A. In other words, if B is a proper subset of A, then all elements of B are in A but A contains at least one element that is not in B.

Important

- *Every set is a subset of itself. i.e., $A \subset A$.*
- *ϕ is a subset of every set.*
- *The total number of subsets of a set is 2^n, where n is the number of elements contained in a set.*

  E.g. Let $A = \{a, b, c\}$, $n(A) = 3$, then total number of subsets of set $A = 2^3 = 8$. The subsets as follows:

  ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{b, c\}$, $\{a, c\}$, $\{a, b, c\}$
- *The number of proper subsets of A is one less than the number of elements in the power the set i.e $2^n - 1$ set itself will not be counted in the proper subset.*

Example 1.8: Let $A = \{1, 2, \{3, 4\}, 5\}$. Which of the following statements are incorrect and why?

(A) $\{3, 4\} \subset A$ (B) $\{\{3, 4\}\} \subset A$

(C) $1 \subset A$ (D) $\{1, 2, 5\} \subset A$

(E) $\{1, 2, 3\} \subset A$ (F) $\phi \in A$

(G) $\phi \subset A$ (H) $\{\phi\} \subset A$ [NCERT]

Ans. Given, set $A = \{1, 2, \{3, 4\}, 5\}$

(A) The statement $\{3, 4\} \subset A$ is incorrect because $3 \in \{3, 4\}$; however, $3 \notin A$.

(B) The statement $\{\{3, 4\}\} \subset A$ is correct because $\{3, 4\} \in \{\{3, 4\}\}$ and $\{3, 4\} \in A$.

(C) The statement $1 \subset A$ is incorrect because an element of a set can never be a subset of itself.

(D) The statement $\{1, 2, 5\} \subset A$ is correct because each element of $\{1, 2, 5\}$ is also an element of A.

(E) The statement $\{1, 2, 3\} \subset A$ is incorrect because $3 \in \{1, 2, 3\}$; however, $3 \notin A$.

(F) The statement $\phi \in A$ is incorrect because ϕ is not an element of A.

(G) The statement $\phi \subset A$ is correct because ϕ is a subset of every set.

(H) The statement $\{\phi\} \subset A$ is incorrect because $\phi \in \{\phi\}$; however, $\phi \notin A$

Example 1.9: Write down all the subsets of the following sets:

(A) $\{a\}$ **(B) $\{1, 2, 3\}$**

(C) $\{-1, 0, 1\}$ **(D) ϕ**

Ans. (A) Subsets of $\{a\}$ are ϕ, $\{a\}$.

(B) Subsets of $\{1, 2, 3\}$ are ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{2, 3\}$, $\{3,1\}$, $\{1, 2, 3\}$.

(C) Subsets of $\{-1, 0, 1\}$ are ϕ, $\{-1\}$, $\{0\}$, $\{1\}$, $\{-1, 0\}$, $\{0,1\}$, $\{-1,1\}$, $\{-1, 0, 1\}$.

(D) Since, ϕ is a subset of every set, hence, the subset of ϕ is ϕ itself.

Subsets of Set of Real Numbers

Some important subsets of real numbers are as under:

(i) The set of natural numbers $N = \{1, 2, 3, ...\}$.

(ii) The set of integers $Z = \{... -3, -2, -1, 0, 1, 2, 3, ...\}$.

(iii) The set of a rational number

$$Q = \left\{x : x = \frac{p}{q}, \text{where } p, q \in Z \text{ and } q \neq 0\right\}$$

(iv) The set of irrational numbers $T = \{x : x \in R \text{ and } x \notin Q\}$, i.e., all real numbers that are not rationals.

Thus, the obvious relations among these subsets are:

$$N \subseteq Z \subseteq Q \subseteq R$$

Intervals as Subsets of *R*

Let us now discuss the various types of intervals which are subsets of the set R of all real numbers. There are four types of intervals:

Open Interval

Let $a, b \in R$ such that $a < b$. Then, the set of all real numbers between a and b excluding both a and b is called an open interval from a to b. It is denoted by}(a, b).

$$(a, b) = [x : x \in R \text{ and } a < x < b\}.$$

Semi-Open Interval

(1) Let $a, b \in R$ such that $a < b$, then, the set of all real numbers between a and b, including a but excluding b, is called semi-closed interval (left closed-right open) from a to b. It is denoted by $[a, b)$.

$$[a, b) = \{x : x \in R \text{ and } a \le x < b\}.$$

(2) Let $a, b \in R$ such that $a < b$, then, the set of all real numbers between a and b, excluding a but including b, is called semi-open interval (left open-right closed) from a to b. It is denoted by $(a, b]$.

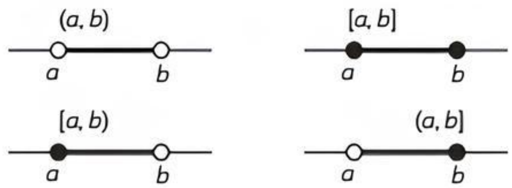
$$(a, b] = [x : x \in R \text{ and } a < x \le b\}.$$

Closed Interval

Let $a, b \in R$ such that $a < b$. Then, the set of all real numbers between a and b including both a and b is called a closed interval from a to b. In other words, the interval which contains the end points is also called a closed interval. It is denoted by $[a, b]$.

$$[a, b] = [x : x \in R \text{ and } a \le x \le b\}.$$

On a real number line, various types of intervals described above as subsets of R, are shown in the figure below.



Important

- *The set $[0, \infty)$ defines the set of non-negative real numbers.*
- *The set $(-\infty, 0)$ defines the set of negative real numbers.*
- *The set $(-\infty, \infty)$ describes the set of real numbers.*

Example 1.10: Write the following as intervals:

(A) $\{x : x \in R, -4 < x \le 6\}$

(B) $\{x : x \in R, -12 < x < -10\}$

(C) $\{x : x \in R, 0 \le x < 7\}$

(D) $\{x : x \in R, 3 \le x \le 4\}$ **[NCERT]**

Ans. (A) $\{x : x \in R, -4 < x \le 6\}$

This set excludes -4 and includes 6, so, this set can be written as a semi-open interval, as $(-4, 6]$.

Hence, $\{x : x \in R, -4 < x \le 6\} = (-4, 6]$.

(B) $\{x : x \in R, -12 < x < -10\}$

This set excludes both -12 and -10, so, this set can be written as an open interval, as $(-12, -10)$.

Hence, $\{x : x \in R, -12 < x < -10\} = (-12, -10)$.

(C) $\{x : x \in R, 0 \le x < 7\}$.

This set includes 0 and excludes 7, so, this set can be written as a semi-closed interval, as [0, 7).

Hence, $\{x: x \in R, 0 \le x < 7\} = [0, 7)$.

(D) $\{x: x \in R, 3 \le x \le 4\}$

This set includes both 3 and 4, so, this set can be written as a closed interval, as [3, 4].

Hence, $\{x : x \in R, 3 \le x \le 4\} = [3, 4]$.

Universal Set

A set that contains elements of all sets under consideration is called a universal set. It is denoted by U. For example, for the set of all integers, the universal set can be the set of rational numbers or, for that matter, the set R of real numbers.

Illustration: Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6\}$, $C = \{1, 3, 5, 7\}$, the universal set for these set can be

$U = \{1, 2, 3, 4, 5, 6, 7\}$

Example 1.11: Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$ which of the following may be considered as universal set (s) for all the three sets A, B and C?

(A) {0, 1, 2, 3, 4, 5, 6}

(B) ϕ

(C) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(D) {1, 2, 3, 4, 5, 6, 7, 8}. [NCERT]

Ans. Given sets are $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$

(A) {0, 1, 2, 3, 4, 5, 6}

Elements of A, B and C together

= {1, 3, 5, 2, 4, 6, 0, 8}

The given set does not contain 8, so it cannot be a universal set for A, B and C.

(B) ϕ

Elements of A, B and C together = {1, 3, 5, 2, 4, 6, 0, 8}

Since, ϕ does not contain any element, so, it cannot be a universal set for A, B and C.

(C) {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Elements of A, B and C together = {1, 3, 5, 2, 4, 6, 0, 8}

Since, the given set contains all the elements of set A, B and C, so it is a universal set for A, B and C.

(D) {1, 2, 3, 4, 5, 6, 7, 8}

Elements of A, B and C together = {1, 3, 5, 2, 4, 6, 0, 8}

Since, the given set does not contain 0, so, it cannot be a universal set for A, B and C.

TOPIC 4

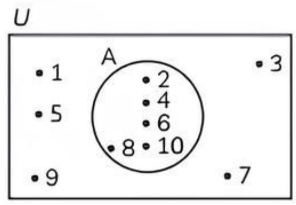
VENN DIAGRAM AND OPERATIONS ON SETS

Venn Diagrams

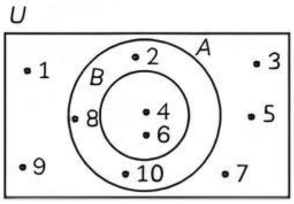
Most of the relationships between sets can be represented by means of diagrams which are known as Venn diagram. A Swiss mathematician, named Euler gave an idea to represent a set by the points in a closed curve. Later on, British mathematician John-Venn (1834- 1883) brought this idea to practice. That is why the diagrams drawn to represent sets are called Venn-Euler diagrams or simply Venn diagrams. These diagrams consist of rectangles and closed curves, usually circles. The universal set is represented usually by a rectangle and its subsets by circles. In Venn diagrams, the elements of the sets are written in their respective circles. Sometimes pictures are very helpful for our thinking.

In Venn diagrams, the universal set U is represented by points within a rectangle and its subsets are represented by points in closed curves (usually circles) within the rectangle and the elements of the sets are written in their respective circles. Also, the elements of a set are within the circle of the set and represented by a point.

Illustration: Let $U = \{1, 2, 3, ..., 10\}$ is an universal set of which $A = \{2, 4, 6, 8, 10\}$ is a subset, its Venn diagram is shown as:



If $B = \{4, 6\}$ is another set such that $B \subset A$, then its Venn diagram can be drawn as



Operations on Sets

We are already familiar with the fundamental operations (addition, subtraction, multiplication, and division) on numbers. Each one of these operations was performed on a pair of numbers to get another number. Similarly, there are some operations that when performed on two sets give rise to another set. We will now define certain operations on sets and examine their properties. Henceforth, we will refer to all our sets as subsets of some universal set.

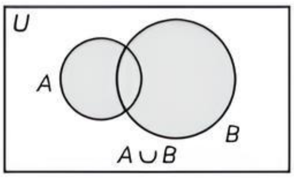
For example, suppose you live in a hostel. You, with your new roommate, decide to have a party in your hostel room, and you both invite your friends. At this party, two sets are being combined, though it might turn out that there are some friends that were in both parties or sets.

Union of Sets

Let A and B be any two sets. Then, the set which consists of all the elements of A and all the elements of B, the common elements being taken only once is known as union of A and B. The symbol '$\cup$' is used to denote the union.

Symbolically, we write the union of A and B as $A \cup B$ and reads as 'A union B', also, we can write $A \cup B$ as $\{x : x \in A \text{ or } x \in B\}$

The union of two sets can be represented by a Venn diagram as



In the above Venn diagram, the shaded portion represents $A \cup B$.

Some properties of the union operations are:

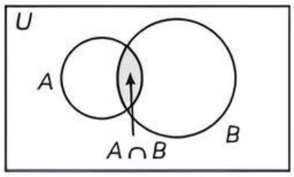
(i)	$A \cup B = B \cup A$	(Commutative law)
(ii)	$(A \cup B) \cup C = A \cup (B \cup C)$	(Associative law)
(iii)	$A \cup \phi = A$	(Law of identity element, ϕ is the identity of $\cup$)
(iv)	$A \cup A = A$	(Idempotent law)
(v)	$U \cup A = U$	(Law of U)

Intersection of Sets

Let A and B be any two sets. Then, the intersection of sets A and B is the set of all elements that are common to both A and B. The symbol '$\cap$' is used to denote the intersection.

We write the intersection of sets A and B as $A \cap B$ and read as 'A intersection B', also we can write $A \cap B$ as $\{x : x \in A \text{ and } x \in B\}$

The intersection of two sets can be represented by a Venn diagram as,



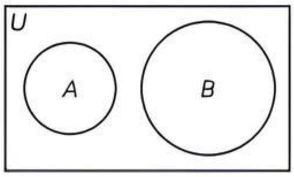
The shaded portion in the above diagram represents $A \cap B$.

If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

For example, let $A = \{2, 4, 6, 8\}$ and $B = \{1, 3, 5, 7\}$.

Clearly, there are no common elements in both sets A and B, which means $A \cap B = \phi$

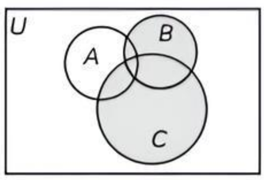
Then, sets A and B are disjoint sets, because there are no elements. Disjoint sets are represented in the Venn diagram as shown in diagram below.



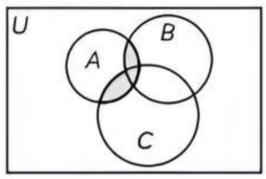
Let A, B and C be the three sets, then some important properties of intersection are:

(i)	$A \cap B = B \cap A$	(Commutative law)
(ii)	$(A \cap B) \cap C = A \cap (B \cap C)$	(Associative law)
(iii)	$\phi \cap A = \phi$, $U \cap A = A$	(Law of ϕ and U)
(iv)	$A \cap A = A$	(Idempotent law)
(v)	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	(Distributive law) i.e., $\cap$ distributes over $\cup$

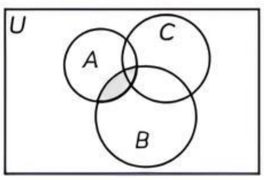
These intersection properties can be represented in Venn diagram as follows:



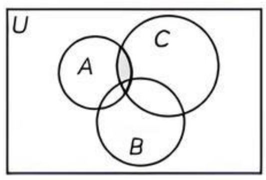
(i) $(B \cup C)$



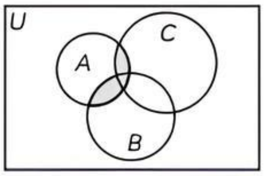
(ii) $A \cap (B \cup C)$



(iii) $(A \cap B)$



(iv) $(A \cap C)$



(v) $(A \cup B) \cup (A \cup C)$

Difference of Sets

Let A and B be any two sets. Then, the set of all those elements of set A that are not in set B, is called the difference of the sets A and B.

Symbolically, we write the difference of A and B as $A - B$ and read as 'A minus B', also, we can write $A - B = \{x : x \in A \text{ and } x \notin B\}$

Important

- *For any two sets A and B, we have* $A - B = A \cap B'$.
- *For any two sets A and B, we have* $B - A = B \cap A'$.

Example 1.12: If $A = \{1, 2, 3, 4\}$ $B = \{3, 4, 5, 6\}$, $C = \{5, 6, 7, 8\}$ and $D = \{7, 8, 9, 10\}$, find:

(A) $A \cup B$ (B) $A \cup C$
(C) $B \cup C$ (D) $B \cup D$
(E) $A \cup B \cup C$ (F) $A \cup B \cup D$
(G) $B \cup C \cup D$ [NCERT]

Ans. (A) $A \cup B = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$
$= \{1, 2, 3, 4, 5, 6\}$

(B) $A \cup C = \{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(C) $B \cup C = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$
$= \{3, 4, 5, 6, 7, 8\}$

(D) $B \cup D = \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

(E) $A \cup B \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

(F) $A \cup B \cup D = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\} \cup \{7, 8, 9, 10\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(G) $B \cup C \cup D = \{3, 4, 5, 6\} \cup \{5, 6, 7, 8\} \cup \{7, 8, 9, 10\}$
$= \{3, 4, 5, 6, 7, 8, 9, 10\}$

Example 1.13: Find the intersection of each pair of sets:

(A) $X = \{1, 3, 5\}$ $Y = \{1, 2, 3\}$
(B) $A = \{a, e, i, o, u\}$ $B = \{a, b, c\}$
(C) $A = \{x : x$ is a natural number and multiple of 3$\}$
$B = \{x : x$ is a natural number less than 6$\}$
(D) $A = \{x : x$ is a natural number and $1 < x \leq 6\}$.
$B = \{x : x$ is a natural number and $6 < x < 10\}$
(E) $A = \{1, 2, 3\}$, $B = \phi$ [NCERT]

Ans. (A) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$
$X \cap Y = \{1, 3, 5\} \cap \{1, 2, 3\}$
$= \{1, 3\}$

(B) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$
$A \cap B = \{a, e, i, o, u\}$. $\cap \{a, b, c\}$
$= \{a\}$

(C) $A = \{x : x$ is a natural number and multiple of 3$\} = \{3, 6, 9 \ldots\}$
$B = \{x : x$ is a natural number less than 6$\}$
$= \{1, 2, 3, 4, 5\}$
$A \cap B = \{3, 6, 9 \ldots\} \cap \{1, 2, 3, 4, 5\}$
$= \{3\}$

(D) $A = \{x : x$ is a natural number and $1 < x \leq 6\}$
$= \{2, 3, 4, 5, 6\}$
$B = \{x : x$ is a natural number and $6 < x < 10\}$
$= \{7, 8, 9\}$
$A \cap B = \{2, 3, 4, 5, 6\} \cap \{7, 8, 9\}$
$= \{ \}$ or ϕ

(E) $A = \{1, 2, 3\}$, $B = \phi$
$A \cap B = \{1, 2, 3\} \cap \{ \}$
$= \phi$

Since, there are no common elements in both sets.

Example 1.14: Which of the following pairs of sets are disjoint?

(A) $\{1, 2, 3, 4\}$ and $\{x : x$ is a natural number and $4 \leq x \leq 6\}$
(B) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$
(C) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$

Ans. (A) Let $A = \{1, 2, 3, 4\}$ and $B = \{x : x$ is a natural number and $4 \leq x \leq 6\} = \{4, 5, 6\}$

We know that, two sets are disjoint if they have no common element.

Here, $A \cap B = \{1, 2, 3, 4\} \cap \{4, 5, 6\} = \{4\} \neq \phi$

Since, there is a common element in both sets.

Hence, the given pair of sets is not disjoint.

(B) Let $A = \{a, e, i, o, u\}$ and $B = \{c, d, e, f\}$

We know that, two sets are disjoint if they have no common element.

Here, $A \cap B = \{a, e, i, o, u\} \cap \{c, d, e, f\}$
$= \{e\} \neq \phi$

Since, there is a common element in both sets.

Hence, the given sets are not disjoint.

(C) Let $A = \{x : x$ is an even integer$\} = \{\ldots, -4, -2, 0, 2, 4, \ldots\}$ and $B = \{x : x$ is an odd integer$\}$ $= \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$

We know that, two sets are disjoint if they have no common element.

Here, $A \cap B = \phi$

Hence, the given pair of sets are disjoint.

Example 1.15: If A and B are two sets such that $A \subset B$, then what is $A \cup B$?

Ans. Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$

Here, every element of set A is in set B. So, A is a subset of B, i.e., $A \subset B$

$$A \cup B = \{1, 2\} \cup \{1, 2, 3\}$$
$$= \{1, 2, 3\}$$

Thus, $A \cup B = B$

Example 1.16: Case Based:

After ending the topic 'Operations on sets', the Maths teacher Mrs. Gargi Sharma of class XI wants to check student's understandings about the topic that she taught, so she writes three sets P, Q and R such that P = {2, 4, 6, 8, 10}, Q = {3, 5, 7, 9} and R = {3, 4, 6, 8, 12} on the blackboard and ask the students of the class observe these sets written on the blackboard and to answer the questions.

Based on the above information, answer the following questions.

(A) Assertion (A): The value of $P \cap Q \neq \phi$

Reason (R): A set that does not contain any element is called the empty set.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

(B) What is the value of $P \cap R$?

(C) Which of the following is correct for two sets P and Q to be disjoint?

(a) $P \cap Q = \phi$ (b) $P \cap Q \neq \phi$

(c) $P \cup Q = \phi$ (d) $P \cup Q \neq \phi$

(D) Which of the following is correct for two sets P and R to be intersecting?

(a) $P \cap R = \phi$ (b) $P \cap R = P$

(c) $P \cap R \neq \phi$ (d) $P \cap R = R$

(E) Find $P \cup Q \cup R$.

Ans. (A) *(d) (A) is false but (R) is true.*

Explanation:

$$P = \{2, 4, 6, 8, 10\}, Q = \{3, 5, 7, 9\}$$
$$P \cap Q = \{2, 4, 6, 8, 10\} \cap \{3, 5, 7, 9\}$$

Here, no elements are common in P and Q.

Therefore, $P \cap Q = \phi$

(B) $P = \{2, 4, 6, 8, 10\}$ and $R = \{3, 4, 6, 8, 12\}$

$$P \cap R = \{2, 4, 6, 8, 10\} \cap \{3, 4, 6, 8, 12\}$$
$$P \cap R = \{4, 6, 8\}$$

(C) *(a)* $P \cap Q = \phi$

Explanation:

There is no common element in P and Q.

Also, we know that two sets are disjoint, if they have no common elements.

Hence, $P \cap Q = \phi$ represent disjoint sets.

(D) *(a)* $P \cap R = \phi$

Explanation: If P and R are intersecting, that means there are some elements common in P and R.

But $P \cap R = \{4, 6, 8\}$ [From B]

So, $P \cap R \neq \phi$

(E) Given, $P = \{2, 4, 6, 8, 10\}$, $Q = \{3, 5, 7, 9\}$ and $R = \{3, 4, 6, 8, 12\}$

$P \cup Q \cup R = \{2, 4, 6, 8, 10\} \cup \{3, 5, 7, 9\} \cup \{3, 4, 6, 8, 12\}$

So, $P \cup Q \cup R = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$

Complement of a Set

Let U be the universal set and A be any subset. Then the complement of A is the set of all elements which are in U but not in A.

We can write

$$A' = \{x : x \in U \text{ and } x \notin A \text{ or } A' = U - A\}$$

Illustration: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 3, 5, 7, 9\}$, then $A' = U - A = \{2, 4, 6, 8, 10\}$

Some Properties of Complement Sets

(i) Complement Law: $A \cup A' = U$ and $A \cap A' = \phi$.

(ii) If A and B are any two subsets of the universal set U, then the complement of the union of two sets is the intersection of their complements and the complement of the intersection of two sets is the union of their complements. $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

These are called De Morgan's laws. These are named after the mathematician De Morgan.

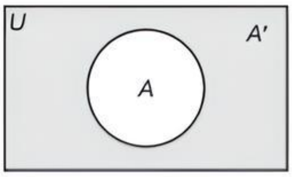
Illustration: Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$ and $B = \{3, 4, 5\}$, then $A' = U - A = \{1, 4, 5, 6\}$, $B' = U - B = \{1, 2, 6\}$, $A \cup B = \{2, 3, 4, 5\}$, $A' \cap B' = \{1, 6\}$ and $(A \cup B)' = U - (A \cup B) = \{1, 6\}$. Hence, $(A \cup B)' = A' \cap B'$

(iii) Law of double complementation: $(A')' = A$.

(iv) Laws of empty set and universal set: $\phi' = U$ and $U' = \phi$.

Important

- *If A is a subset of the universal set U, then its complement A' is also a subset of U.*



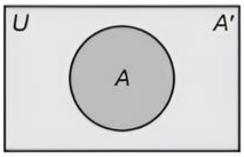
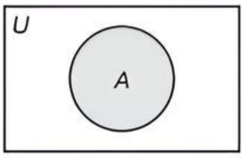
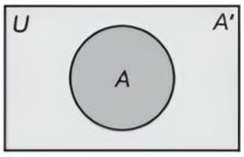
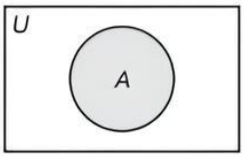
- *The shaded portion represents the complement of set A.*

Example 1.17: Fill in the blanks to make each of the following a true statement:

(A) $A \cup A' = \ldots$ (B) $\phi' \cap A = \ldots$

(C) $A \cap A' = \ldots$ (D) $U' \cap A = \ldots$

Ans.

(A)	$A \cup A' = U$	
(B)	$\phi' = U - \{\ \} = U$ So, $\phi' = U$ Now, $\phi' \cap A = U \cap A = A$	
(C)	$A \cap A' = \phi$	
(D)	$U' \cap A = U' = U - U$ $= \{\ \} = \phi$ So, $U' = \phi$ Thus, $U' \cap A = \phi \cap A = \phi$	

Example 1.18: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, 7\}$. Verify that

(A) $(A \cup B)' = A' \cap B'$

(B) $(A \cap B)' = A' \cup B'$ **[NCERT]**

Ans. (A) To verify that $(A \cup B)' = A' \cap B'$

$A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$
$= \{2, 3, 4, 5, 6, 7, 8\}$

LHS $= (A \cup B)' = U - (A \cup B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 4, 5, 6, 7, 8\}$
$= \{1, 9\}$

Also, $A' = U - A$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$
$= \{1, 3, 5, 7, 9\}$

and $B' = U - B$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 5, 7\}$
$= \{1, 4, 6, 8, 9\}$

Thus, RHS $= A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\}$
$= \{1, 9\}$

Clearly, $(A \cup B)' = A' \cap B'$

Hence, verified.

(B) We have to verify that $(A \cap B)' = A' \cup B'$

$A \cap B = \{2, 4, 6, 8\} \cap \{2, 3, 5, 7\} = \{2\}$

Thus, LHS $= (A \cup B)' = U - (A \cap B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2\}$
$= \{1, 3, 4, 5, 6, 7, 8, 9\}$

$A' = U - A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 6, 8\}$
$= \{1, 3, 5, 7, 9\}$

$B' = U - B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 3, 5, 7\}$
$= \{1, 4, 6, 8, 9\}$

And so, RHS $= A' \cup B' = \{1, 3, 5, 7, 9\} \cup \{1, 4, 6, 8, 9\}$
$= \{1, 3, 4, 5, 6, 7, 8, 9\}$

$(A \cap B)' = A' \cup B'$

Hence, verified.

Example 1.19: Taking the set of natural numbers as the universal set, write down the complements of the following sets:

(A) $\{x : x$ is a prime number$\}$

(B) $\{x : 2x + 5 = 9\}$

(C) $\{x : x \in N$ and $2x + 1 > 10\}$ **[NCERT]**

Ans. (A) We have to find complements of $\{x : x$ is a prime number$\}$

Natural Number

Prime numbers	Composite numbers	Neither prime nor composite
Eg. 2, 3, 7, 11, 13	*Eg.* 4, 6, 8, 9, 10, 12	*Eg.* 1

U is set of natural numbers

$\{x : x$ is a prime number$\} = \{x : x \in N$ and x is a composite number or $x = 1\}$

(B) We have to find complements of $\{x : 2x + 5 = 9\}$

$U = \{1, 2, 3, 4, 5, \ldots\}$

$2x + 5 = 9$

$2x = 9 - 5$

$2x = 4$

$x = \frac{4}{2}$

$x = 2$

$\{x : 2x + 5 = 9\}' = \{x : x = 2\}'$

$= \{x : x \in N \text{ and } x \neq 2\}$

(C) We have to find complements of $\{x : x \in N$ and $2x + 1 > 10\} = \{x : x \in N \text{ and } 2x + 1 > 10\}'$

$U = \{1, 2, 3, 4, 5, \ldots\}$

$2x + 1 > 10$

$2x > 10 - 1$

$2x > 9$

$x > \frac{9}{2}$

$x > 4.5$

i.e. $\{x : x \in N$ and $2x + 1 > 10 = \{x : x \in N$ and $x > 4.5\}$

Thus, $\{x : x \in$ N and $2x + 1 > 10 = \{x : x \in N$ and $x > 4.5\}'$

$= \{x : x \in N \text{ and } x \leq 4.5\}$

Example 1.20: Show that for any sets A and B,

(A) $A = (A \cap B) \cup (A - B)$

(B) $A \cup (B - A) = (A \cup B)$

Ans. (A) $A = (A \cap B) \cup (A - B)$

Consider RHS $= (A \cap B) \cup (A - B)$

$(A \cap B) \cup (A - B')$

[by def of difference of sets, $A - B = A \cap B'$]

$= A \cap (B \cup B')$ [by distributive]

$= A \cap U$ [$\because A \cup A' = U$]

$= A$

$=$ LHS

Hence, $A = (A \cap B) \cup (A - B)$

(B) $A \cup (B - A) = A \cup B$

Consider, $A \cup (B - A)$

$= A \cup (B \cap A')$

[by def of difference of sets, $A - B = A \cap B'$]

$= (A \cup B) \cap (A \cup A')$ [by distributive property]

$= (A \cup B) \cap U$ [$\because A \cup A' = U$]

$= A \cup B$

Example 1.21: In each of the following, determine whether the statement is true or false. If it is true, prove it. If it is false, give an example.

(A) If $A \subset B$ and $B \in C$, then $A \in C$

(B) If $A \subset B$ and $B \subset C$, then $A \subset C$ [NCERT]

Ans. (A) If $A \subset B$ and $B \in C$, then $A \in C$

Let $A = \{2\}$.

Since, $A \subset B$, element of set A i.e., 2 should be an element of set B taking $B = \{0, 2\}$

Also, since $B \in C$, i.e., whole set B is an element of set C

Taking $C = \{1, \{0, 2\}, 3\}$

We have to prove that $A \in C$

But, $\{2\} \notin C$ as $\{2\}$ is not element of C

Hence, $A \notin C$

So, the given statement is false.

(B) If $A \subset B$ and $B \subset C$, then $A \subset C$

Let $A = \{1\}$.

Since, $A \subset B$, all elements of set A i.e., 1 should be an element of set B

Taking $B = \{1, 2\}$

Also, $B \subset C$, all elements of set B i.e., 1, 2 should be an element of set C

Hence, taking $C = \{1, 2, 3\}$

We have to prove that $A \subset C$

Since all elements of A i.e., 1 is in set C, A is a subset of C

So, the given statement is true.

Example 1.22: Show that the following four conditions are equivalent:

(A) $A \subset B$ **(B) $A - B = \phi$**

(C) $A \cup B = B$ **(D) $A \cap B = A$**

Ans. Showing condition (A) is equivalent to condition (B)

If $A \subset B$, this means that all elements of A are in B,

So, A has no elements different from B

$\Rightarrow A - B = \phi$

Showing condition (B) is equivalent to condition (C)

$A - B = \phi$

This means A has no elements different from B

So, all elements of A are in B

So, $A \cup B = B$

Showing condition (C) is equivalent to condition (D)

$A \cup B = B$

This means all elements of A are in B.

So, the common elements of A and B must be the elements of A

So, $A \cap B = A$

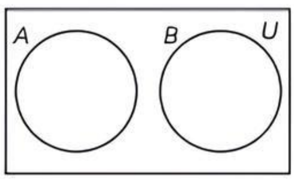
Thus, (A) $\Leftrightarrow$ (B) $\Leftrightarrow$ (C) $\Leftrightarrow$ (D)

Thus, all the four conditions are equivalent.

TOPIC 5

APPLICATIONS OF SETS

If A and B are two finite sets, then there are two cases that may arise.

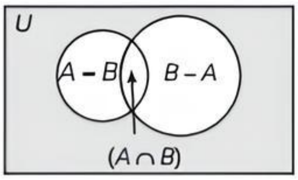


Case I: If A and B are disjoint sets then, there will be no common element in A and B, i.e., $A \cap B = \phi$

Thus, $n(A \cup B) = n(A) + n(B)$

Case II: If A and B are not disjoint sets then, there will be common element in A and B, then

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$



Caution

↝ *Students should understand that if A and B are disjoint sets, then the formula will be written as*

$n(A \cup B) = n(A) + n(B)$ because for disjoint set $A \cap B = \phi$

Some Important results

(1) If A and B are finite sets and $A \cap B = \phi$, then $n(A \cup B) = n(A) + n(B)$

(2) If A and B are non-disjoint finite sets, then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(3) We can see from the above Venn diagram that the sets $A - B$, $A \cap B$ and $B - A$ are disjoint and their union is $A \cup B$, therefore, we can write,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

We can also write,

$$n(A) = n(A - B) + n(A \cap B)$$
$$n(B) = n(B - A) + n(A \cap B)$$

(4) If A, B and C are finite sets, and number of elements of three sets are given, then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - (A \cap C) + n(A \cap B \cap C)$$

In the following table, we give some verbal descriptions with their equivalent set-theoretic notation.

Verbal Description	Theoretical Notation
For two sets problems:	
Not A	A'
Only A	$A \cap B'$
A but not B	$A \cap B'$
Only B	$A' \cap B$
B but not A	$A' \cap B$
Either A or B	$A \cup B$
At least one of A and B	$A \cup B$
A and B	$A \cap B$
A or B	$A \cup B$
Neither A nor B	$A' \cap B'$
For three sets problems:	
Not A	A'
Only A	$A \cap B' \cap C'$
Only B	$A' \cap B \cap C'$
Only C	$A' \cap B' \cap C$
Only A and B	$A \cap B \cap C'$
Only B and C	$A' \cap B \cap C$
Only A and C	$A \cap B' \cap C$
At least one of A, B, and C	$A \cup B \cup C$
None of A, B, and C	$A' \cap B' \cap C$

Example 1.23: If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have? [NCERT]

Ans. Given, $n(X \cup Y) = 18$

$n(X) = 8$

$n(Y) = 15$

We know that,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$
$$18 = 8 + 15 - n(X \cap Y)$$
$$18 = 23 - n(X \cap Y)$$
$$n(X \cap Y) = 23 - 18$$
$$n(X \cap Y) = 5$$

Example 1.24: In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many people only like tennis? [NCERT]

Ans. Let C and T denote the set of people who like cricket and tennis, respectively.

Number of people in the group

= Number of people who like cricket or tennis

$= n(C \cup T) = 65$

Number of people who like cricket $= n(C) = 40$.

Number of people who like both cricket and tennis $= n(C \cap T) = 10$

We know that,

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$
$$65 = 40 + n(T) - 10$$
$$65 = 40 - 10 + n(T)$$
$$65 = 30 + n(T)$$
$$65 - 30 = n(T)$$
$$n(T) = 35$$

Therefore, 35 people like tennis (i.e., who like only tennis and also those who like both tennis and cricket, as depicted below)

People who like tennis

↙ People who like only tennis

↘ People who like both tennis and cricket

Number of people who like only tennis but not cricket

= Number of people who like tennis – Number of people who like both tennis and cricket

$= n(T - C)$
$= n(T) - n(T \cap C)$
$= 35 - 10$
$= 25$

Thus, 25 people only like tennis.

Example 1.25: **If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?** **[NCERT]**

Ans. Given

$$n(X) = 40, n(X \cup Y) = 60, n(X \cap Y) = 10$$

We know that,

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$
$$60 = 40 + n(Y) - 10$$
$$60 = 40 - 10 + n(Y)$$
$$60 = 30 + n(Y)$$
$$\therefore \quad n(Y) = 60 - 30 = 30$$

Thus, the set Y has 30 elements.

Example 1.26: **In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked product C only.** **[NCERT]**

Ans. Let A, B and C be the sets of people who like product A, product B and product C respectively.

Number of people who liked product A
$= n(A) = 21$,

Number of people who liked product B
$= n(B) = 26$,

Number of people who liked product C
$= n(C) = 29$,

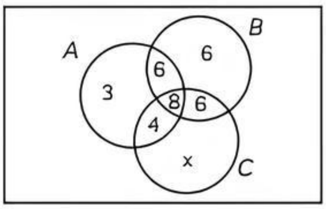
Number of people who liked product A and B
$= n(A \cap B) = 14$

Number of people who liked product C and A
$= n(C \cap A) = 12$

Number of people who liked product B and C
$= n(B \cap C) = 14$

Number of people who liked all three products A, B and C
$= n(A \cap B \cap C) = 8$

Let us draw a Venn diagram



Number of people who liked product C only

$$= n(C) - n(C \cap A) - n(B \cap C) + n(A \cap B \cap C)$$
$$= 29 - 12 - 14 + 8 = 11$$

Hence, the number of people who like product C only is 11.

Example 1.27: **Case Based:**

In a school, during the new academic session 2021-2022, in all sections of class XI, out of 200 students, 30 students offered English only, 24 students offered Sanskrit only, 16 students offered only Hindi, 80 students offered Hindi and English, 40 students offered Hindi and Sanskrit, 20 students offered English and Sanskrit, 130 students offered Hindi.

Based on the above information answer the following questions:

(A) Assertion (A): The number of students who offered all the three subjects are 7.

Reason (R): The diagrams which are drawn to represent sets are called Venn diagrams.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

(B) The number of students who offered English is:

(a) 130 (b) 120
(c) 124 (d) 100

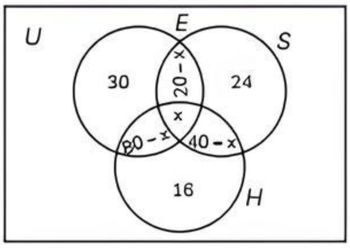
(C) The number of students who offered Sanskrit is:

(a) 56 (b) 62
(c) 75 (d) 78

(D) Find the number of students who offered English and Sanskrit but not Hindi.

(E) Find the number of students who did not offer any of the above three subjects.

Ans. Let E, S and H be the sets of students who offered English, Sanskrit and Hindi, respectively. Let x be the number of students who offered all the three subjects, then the number of members in different regions are shown in the following diagram.



(A) *(d)* *(A) is false but (R) is true.*

Explanation:

From the above Venn diagram,

We have, the number of students who offered Hindi.

$(80 - x) + x + (40 - x) + 16 = 130$ [Given]

$\Rightarrow \quad 136 - x = 130$

$\Rightarrow \quad x = 6$

The number of students who offered all the three subjects are 6.

(B) *(c) 124*

Explanation:

From the above Venn diagram,

The number of students who offered English

$= 30 + (20 - x) + x + (80 - x)$

$= 130 - x$

$= 130 - 6$

$= 124$ [$\because x = 6$, from (A)]

(C) *(d) 78*

Explanation:

From the above Venn diagram,

The number of students who offered Sanskrit

$= 24 + (20 - x) + x + (40 - x)$

$= 84 - x$

$= 84 - 6 = 78$ [$\because x = 6$, from (A)]

(D) From the above Venn diagram,

The number of students who offered English and Sanskrit but not Hindi

$= 20 - x = 20 - 6 = 14$

[$\because x = 6$, from (A)]

(E) The number of students who offered any of the three subjects

$= 30 + 24 + 16 + (20 - x) + (80 - x) + (40 - x) + x$

$= 210 - 2x$

$= 210 - 2 \times 6 = 198$ [$\because x = 6$, from (A)]

The number of students who did not offer any one of the three subjects $= 200 - 198 = 2$

OBJECTIVE Type Questions

[**1** mark]

Multiple Choice Questions

1. **If $X = \{8^n - 7n - 1 | n \in N\}$ and $Y = \{49n - 49 | n \in N\}$, then:**

(a) $X \subset Y$ **(b)** $Y \subset X$

(c) $X = Y$ **(d)** $X \cap Y = \phi$

[NCERT Exemplar]

Ans. *(a)* $X \subset Y$

Explanation:

$X = \{8^n - 7n - 1 | n \in N\} = \{0, 49, 490, ...\}$

$Y = \{49n - 49 | n \in N\} = \{0, 49, 98, 147, ... 490, ...\}$

Clearly, every element of X is in Y but every element of Y is not in X.

$\therefore \quad (X \subset Y)$

2. **The roster form of the following set is a set of integers between -5 and 5 is:**

(a) $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

(b) $\{-5, -4 ... 4, 5\}$

(c) $\{0, 1, 2, 3, 4, 5\}$

(d) none of these

Ans. *(a)* $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

Explanation: According to the question, we have to write the integers that come in between –5 and 5 means –5, 5 are not included, so the roster form is $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$.

3. **Which of the following set is a finite set ?**

(a) Set of concentric circles

(b) Set of letters in English alphabets

(c) $x \in R : a < x > 1\}$

(d) None of these

Ans. *(b) Set of letters in English alphabets*

Explanation: From the above options, only option (b) contains countable elements, as we can count the elements of a set of letters in English alphabets, hence it is a finite set .

4. **If $X = \{1, 2, 3\}$, if n represents any member of x, then all elements of a set, containing element $n + 6$ is given by:**

(a) $\{6, 7, 8\}$ **(b)** $\{5, 6, 7\}$

(c) $\{7, 8, 9\}$ **(d)** $\{8, 9, 10\}$

Ans. *(c)* $\{7, 8, 9\}$

Explanation: The elements in a set containing $n + 6$ elements where $n \in x$ will

$1 + 6, 2 + 6, 3 + 6 = \{7, 8 \ 9\}$.

5. **Let S be an infinite set and $S_1, S_2, S_3, ..., S_n$ be sets such that $S_1 \cup S_2 \cup S_3 \cup S_n = S$ then**

(a) at least one of the sets S_i is a finite set

(b) not more than one of the set S_i can be infinite

(c) at least one of the sets S_i is an infinite set

(d) none of these [Diksha]

Ans. *(a) at least one of the sets S_i is a finite set*

Explanation: $S_1 \cup S_2 \cup S_3 \cup S_n$. For S to be an infinite set, at least one of sets S_i must be infinite, since if all S_i were finite, then S will also be finite.

6. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of second set. The values of m and n are respectively:

(a) 4, 7 (b) 7, 4

(c) 4, 4 (d) 7, 7

[Diksha, Delhi Gov. SQP 2022]

Ans. *(b) 7, 4*

Explanation: Given that the total number of subsets of the first set is 112 more than the total number of subsets of the second set.

$\Rightarrow \quad 2^m - 2^n = 112$

$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^4 \times 7$

$\therefore\ 2^n = 2^4$ and $(2^{m-n} - 1) = 7$

We can say that the value of $n = 4$

$\Rightarrow \quad 2^{m-n} - 1 = 7$

$\Rightarrow \quad 2^{m-n} = 8$

$\Rightarrow \quad 2^{m-4} = 2^3\ [\because n = 4]$

$\Rightarrow \quad m - 4 = 3$

$\Rightarrow \quad m = 7$

Therefore the value of $m = 7$ and $n = 4$.

7. The symmetric difference of $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$ is:

(a) {1, 2} (b) {1, 2, 4, 5}

(c) {4, 3} (d) {2, 5, 1, 4, 3}

[Diksha]

Ans. *(b) {1, 2, 4, 5}*

Explanation: $A \Delta B = (A - B) \cup (B - A)$

$A - B = \{1, 2\}$

$B - A = \{4, 5\}$

So, $(A - B) \cup (B - A) = \{1,2,4,5\}$

8. Let $A = \{x : x \in R, x > 6\}$ and $B = \{x \in R : x < 9\}$. Then, $A \cap B$ is equal to:

(a) (7, 8] (b) (7, 8)

(c) [7, 8) (d) [7, 8]

Ans. *(b) (7, 8)*

Explanation:

$A = \{x : x \in R, x > 6\}$

$= A = \{7, 8, 9, \ldots\}$,

$B = \{x \in R : x < 9\}$

$= \{8, 7, 6, 5, \ldots\}$

$A \cap B = \{x : x \in R, x > 6\} \cap \{x \in R : x < 9\}$

$= \{x : x \in R, x > 6 \text{ and } x < 9\}$

$= \{x : x \in R, 6 < x < 9\}$

(it shows an open interval)

$= (7, 8)$

9. Let U be the universal set containing 700 elements. If A, B are subsets of $\cap$ such that $n(A) = 300$, $n(B) = 400$ and $n(A \cap B) = 50$. Then, $n(A' \cap B')$ is equal to:

(a) 400 (b) 650

(c) 300 (d) none of these

Ans. *(d) none of these*

Explanation: Given,

$n(A) = 300$, $n(B) = 400$ and $n(A \cap B) = 50$

and $n(U) = 700$

We have,

$n(A' \cap B') = n(A \cup B)'$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 300 + 400 - 50$

$= 650$

$n(A \cup B)' = 700 - 650 = 50$

Thus,

$n(A' \cap B') = 50$

10. Let A and B be two sets such that $n(A) = 20$, $n(B) = 10$, $n(A \cup B) = 15$. Then, $n(A \cap B)$ is equal to:

(a) 30 (b) 40

(c) 15 (d) none of these

Ans. *(c) 15*

Explanation: Given, A and B are two sets such that

$n(A) = 20$, $n(B) = 10$, $n(A \cup B) = 15$

We know that,

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$15 = 20 + 10 - n(A \cap B)$

$15 = 30 - n(A \cap B)$

$n(A \cap B) = 15$

11. If A and B are two disjoint sets, then $n(A \cap B)$ is equal to:

(a) 0

(b) $n(A) + n(B) - n(A \cap B)$

(c) $n(A) + n(B) + n(A \cap B)$

(d) $n(A)\, n(B)$

Ans. *(a) 0*

Explanation: If A and B are two disjoint sets, then there will be no common elements set A and B.

Thus, $A \cap B = \phi$

Hence, $n(A \cap B) = 0$

12. If A and B are two sets such that $n(A) = 50$, $n(B) = 20$, $n(A \cap B) = 40$, then $n(A \cup B)$ is equal to:

(a) 20 (b) 30

(c) 40 (d) 50

Ans. *(b) 30*

Explanation: Given, A and B are two sets such that $n(A) = 50$, $n(B) = 20$, $n(A \cap B) = 40$.

We have,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Putting the values, we get

$$n(A \cup B) = 50 + 20 - 40$$

$$n(A \cup B) = 30$$

13. **Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively.**

(a) 4, 7 (b) 7, 4

(c) 4, 4 (d) 7, 7

[NCERT Exemplar]

Ans. *(b) 7, 4*

Explanation: Given that m and n are elements of two finite sets.

Number of subsets of first set = 2^m

Number of subsets of second set = 2^n

According to question,

$$2^m - 2^n = 112 = 2^4 \times 7$$

$$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^4 (2^3 - 1)$$

On comparing, we get

$$n = 4 \text{ and } m - n = 3$$

$$\Rightarrow \quad n = 4 \text{ and } m = 7$$

14. **If $S = \{ x \mid x$ is a positive multiple of 3 less than 100$\}$ and $P = \{ x \mid x$ is a prime number less than 20$\}$. Then, $n(S) + n(P)$ is equal to:**

(a) 34 (b) 31

(c) 33 (d) 41

[NCERT Exemplar]

Ans. *(d) 41*

Explanation: $S = \{x$: x is a positive multiple of 3 less than 100$\}$

$$\Rightarrow \quad S = \{ 3, 6, 9, 12, \ldots, 99\}$$

$$\Rightarrow \quad n(S) = 33$$

$P = \{x$: x is a prime number less than 20$\}$

$$\Rightarrow \quad P = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

$$n(P) = 8$$

$$n(S) + n(P) = 33 + 8$$

$$= 41$$

15. **In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Then the number of persons who read neither is:**

(a) 210 (b) 290

(c) 180 (d) 260

[NCERT Exemplar]

Ans. *(b) 290*

Explanation: Let H be the set of persons who read Hindi and E be the set of persons who read English.

Then

$$n(U) = 840, n(H) = 450, n(E) = 300$$

$$n(H \cap E') = 200$$

Number of persons who read neither

$$= n(H' \cap E')$$

$$= n(H \cap E)' = n(U) - n(H \cup E)$$

$$= 840 - [n(H) + n(E) - n(H \cap E)]$$

$$= 840 - (450 + 300 - 200)$$

$$= 840 - 550$$

$$= 290$$

16. **If $A = \{x : x$ is a multiple of 3$\}$ and, $B = \{x : x$ is a multiple of 5$\}$, then $A \cap B$ is:**

(a) $\{ x : x$ is a multiple of 3$\}$

(b) $\{ x : x$ is a multiple of 5$\}$

(c) $\{ x : x$ is a multiple of 15$\}$

(d) none of the above

Ans. *(c) $\{x : x$ is a multiple of 15$\}$*

Explanation:

$$A = \{3, 6, 9, 12, 15, 18, 21, \ldots\}$$

$$B = \{5, 10, 15, 20, 25, 30, \ldots\}$$

Then,

$$A \cap B = \{3, 6, 9, 12, 15, 18, 21, \ldots\} \cap \{5, 15, 20, 5, 30 \ldots\}$$

$$= \{15, 30, 45, \ldots\} = \{x : x \text{ is a multiple of } 15\}$$

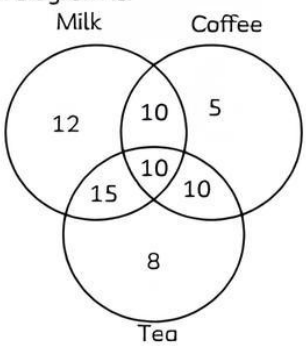
17. **An investigator interviewed 100 students to determine the performance of three drinks: milk, coffee, and tea. The investigator reported that 10 students take all three drinks milk, coffee, and tea; 20 students take milk and coffee; 25 students take milk and tea; 20 students take coffee and tea; 12 students take milk only; 5 students take coffee only and 8 students take tea only. Then the number of students who did not take any of the three drinks is**

(a) 10 (b) 20

(c) 25 (d) 30

Ans. *(d) 30*

Explanation: According to the given information, the Venn diagram is:



Now, $n(M \cup C \cup T) = 12 + 5 + 8 + 10 + 15 + 10 + 10 = 70$

Now, number of people who did not take any of the drinks is $n(M' \cap C' \cap T') = n(M \cup C \cup T)'$

$\Rightarrow \quad n(M \cup C \cup T)' = N - n(M \cup C \cup T)$

$\Rightarrow \quad n(M \cup C \cup T)' = 100 - 70 = 30$

18. In a class of 175 students, the following data shows the number of students opting for one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemistry 23; Mathematics, Physics and Chemistry 18. How many students have been offered Mathematics alone?

(a) 35 (b) 48

(c) 60 (d) 22

Ans. *(c) 60*

Explanation: Let *M*, *P* and *C* denote the set of students who opt for Mathematics, Physics and Chemistry respectively.

Number of students who opted Mathematics only

$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$

$= 100 - 30 - 28 + 18 = 60$

19. In a room containing 28 people, there are 18 people who speak English, 15 people who speak Hindi and 22 people speak Kannada. 9 people speak English and Hindi, 11 people speak both Hindi and Kannada and 13 people who speak both English and Kannada. Find the number of people who speak all the 3 languages.

(a) 6 (b) 7

(c) 8 (d) 9 [Diksha]

Ans. *(a) 6*

Explanation:

Total number of people = 28

Number of people who speak English = 18

Number of people who speak Hindi = 15

Number of people who speak Kannada = 22

Number of people who speak both English and Hindi = 9

Number of people who speak both Hindi and Kannada = 11

Number of people who speak both English and Kannada = 13

Let, people who speak all 3 languages be *x*

Total number of people = number of people speaking (Hindi + English + Kannada) – number of people speaking (Hindi and English + English and Kannada + Kannada and Hindi)+*x*

$28 = (15 + 22 + 18) - (9 + 11 + 13) + x$

$x = 28 - 55 + 33$

$x = 6$

Assertion Reason Questions

Direction: In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R).

Choose the correct answer out of the following choices.

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

20. Assertion (A): The set $D = \{x : x$ is even prime number$\}$ in roster form is $\{2,3\}$.

Reason (R): The set E = the set of all letters in the word: 'SCHOOL', in the roster form is {S, C, H, O, L}.

Ans. *(d) (A) is false but (R) is true.*

Explanation: We can see that 2 is the only even prime number here. So,

$D = \{x : x$ is even prime number$\} = \{2\}$

Thus, the given roster form of set *D* is wrong.

There are 6 letters in the word 'SCHOOL' out of which letter *O* is repeated.

Hence, set *E* in the roster form is {*S*, *C*, *H*, *O*, *L*}.

21. Assertion (A): The set {1, 8, 27,. . ., 1000} in the set-builder form is $\{x : x = n^3$, where $n \in N$ and $1 \le n \le 10\}$.

Reason (R): In roster form, the order in which the elements are listed is immaterial.

Ans. *(b) Both (A) and (R) are true but (R) is not the correct explanation of (A).*

Explanation: We can see that each member in the given set is the cube of a natural number.

Hence, the given set in the set-builder form is $\{x : x = n^3$, where $n \in N$ and $1 \le n \le 10\}$.

Also, in roster form, the order in which the elements are listed is immaterial.

22. Assertion (A): The set {*x* : *x* is a month of a year not having 30 days} in roster form is {January, February, March, May, July, August, October, December}.

Reason(R): A collection of objects is called set.

Ans. *(c) (A) is true but (R) is false.*

Explanation: The months not containing 30 days are January, February, March, May, July, August, October, and December.

So, the roster form of a given set = {January, February, March, May, July, August, October, December}, which is a well-defined collection of months.

R is wrong as mere collection of objects is not a set. the collection should be well defined.

23. **Assertion (A):** The set $A = \{a, e, i, o, u\}$ is a finite set.

Reason (R): Finite set has finite number of elements.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

Explanation: We have a set that is empty or consists of a definite number of elements is called a finite set. Here, set $A = \{a, e, i, o, u\}$ which contains 5 elements. So, it is a finite set.

24. **Assertion (A):** Let $A = \{2, 3, 4\}$ and $B = \{1, 2, 3, 4\}$ Then $A \subset B$

Reason (R): If every element of set A is also an element of set B, then A is a subset of B.

Ans. *(a) Both (A) and (R) are true and (R) is the correct explanation of (A).*

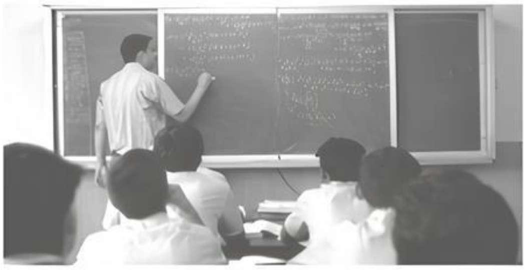
Explanation: Since, every element of A is in B so $A \subset B$.

CASE BASED Questions (CBQs)

[**4** & **5** marks]

Read the following passages and answer the questions that follow:

25. *To check the understanding of sets, a Math teacher writes two sets A and B having finite numbers of elements. The sum of cardinal numbers of two finite sets A and B is 9. The ratio of a cardinal number of the power set of A is to a cardinal number of the power set of B is 8 : 1.*



(A) The cardinal number of set A is:

(a) 2 (b) 3

(c) 6 (d) 8

(B) The cardinal number of set B is:

(a) 2 (b) 3

(c) 6 (d) 8

(C) The maximum value of $n(A \cup B)$ is:

(a) 3 (b) 6

(c) 8 (d) 9

(D) The minimum value of $n(A \cup B)$ is:

(a) 3 (b) 6

(c) 8 (d) 9

(E) If $B \subset A$, then $n(A \cap B)$ is:

(a) 3 (b) 6

(c) 8 (d) 6

Ans. (A) *(c) 6*

Explanation: Let the cardinal numbers of sets A and B be $n(A)$ and $n(B)$ respectively.

Given, $n(A) + n(B) = 9$ ...(i)

Also, the cardinal number of the power set of $A = 2^{n(A)}$

And the cardinal number of the power set of $B = 2^{n(B)}$

Given, $\frac{2^{n(A)}}{2^{n(B)}} = \frac{8}{1}$

$\Rightarrow \quad 2^{n(A)-n(B)} = 2^3$

$\Rightarrow \quad n(A) - n(B) = 3$ ...(ii)

On adding (i) and (ii), we get

$2n(A) = 12$

$\Rightarrow \quad n(A) = 6$

Thus, the cardinal number of set A is 6.

(B) *(b) 3*

Explanation: On subtracting (ii) from (i), we get

$2n(B) = 6$

$\Rightarrow \quad n(B) = 3$

Thus, the cardinal number of set B is 3.

(C) *(d) 9*

Explanation: We have,

$n(A \cup B) = n(A) + n(B) - n(A \cup B)$

The value of $n(A \cup B)$ will be maximum when $n(A \cap B)$ will be minimum.

The minimum value of $n(A \cap B) = 0$.

So, maximum value of

$n(A \cap B) = n(A) + n(B) = 6 + 3 = 9$

(D) *(b)* 6

Explanation: We have,

$n(A \cup B) = n(A) + n(B) - n(A \cup B)$

The value of $n(A \cup B)$ will be minimum when $n(A \cap B)$ will be maximum.

The maximum value of $n(A \cap B) = 3$.

So, minimum value of

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 6 + 3 - 3 = 6$

(E) *(a)* 3

Explanation: Given $B \subset A$

$\Rightarrow A \cap B = B$

$\Rightarrow n(A \cap B) = n(B)$

$\Rightarrow n(A(\cap B) = 3$

26. *In a library, 25 students are reading books on physics, chemistry, and mathematics. It was found that 15 students were reading mathematics, 12 reading physics and 11 reading chemistry, 5 students reading both mathematics and chemistry, 9 students reading both physics and mathematics, 4 students reading both physics and chemistry, and 3 students reading all three subjects.*

(A) Find the number of students reading only Chemistry.

(B) Find the number of students reading only Mathematics.

(C) Find the number of students reading at least one of the subject and also find the number of students reading none of the subjects.

Ans. Let M denote a set of students who are reading mathematics, P denotes who is reading physics and C denotes who is reading chemistry.

We have,

$n(U) = 25, n(M) = 15, n(P) = 12, n(C) = 11$

$n(M \cap C) = 5, n(M \cap P) = 9, n(P \cap C) = 4$

$n(M \cap P \cap C) = 3$

(A) The number of students reading only chemistry

$= n(M' \cap P' \cap C)$

But, $n(M' \cap P' \cap C) = n((M \cap P)' \cap C)$

$= n(C) - n((M \cap P) \cap C)$

[since, $n(A \cap B') = n(A) - n(A \cap B)$]

$= n(C) - n((M \cap C) \cup (P \cap C))$

$= n(C) - n(M \cap C) + n(P \cap C) - n(M \cap P \cap C))$

$= 11 - (5 + 4 - 3) = 5$

(B) The number of students reading only Mathematics $n(M \cap P' \cap C')$

But, $n(M \cap P' \cap C') = n(M \cap (P \cap C)')$

$= n(M) - n(M \cap (P \cap C))$

$= n(M) - n((M \cap P) \cup (M \cap C))$

$= n(M) - (n(M \cap P) + n(M \cap C) - n(M \cap P \cap C))$

$= 15 - (9 + 5 - 3) = 4$

(C) The number of students reading at least one of the subject $= n(M \cup P \cup C)$

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P)$

$- n(P \cap C) - n(M \cap C) + n(M \cap P \cap C)$

$= 15 + 12 + 11 - 9 - 4 - 5 + 3$

$= 41 - 18 = 23$

The number of students reading none of the subjects

$= n(M' \cap P' \cap C') = n(M \cup P \cup C)$

But, $n(M \cup P \cup C)'$

$= n(U) - ((M \cup P \cup C) = 25 - 23 = 2$

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

27. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find $X - Y$ and $Y - X$.

Ans. Given, $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$.

Then, $X - Y = \{a, b, c, d\} - \{f, b, d, g\} = \{a, c\}$

And $Y - X = \{f, b, d, g\} - \{a, b, c, d\} = \{f, g\}$

28. For any two sets A and B, prove that:

(A) $(A - B) \cup B = A \cup B$

(B) $(A - B) \cap B = \phi$

Ans. (A) $(A - B) \cup B = (A \cup B') \cup B$

$= (A \cup B) \cup (B' \cup B)$

[By Distributive Law]

$= (A \cup B) \cup U = A \cup B$

(B) $(A - B) \cap B = (A \cap B') \cap B$

$= A \cap (B' \cap B)$

[By Associative Law]

$= A \cap \phi = \phi$

29. If $U = \{1, 2, \ldots 10\}$, $A = \{1, 2, 3, 5\}$, $B = \{2,4,6,7\}$ and $C = \{2, 3, 4, 8\}$, find

(A) $(B \cup C)'$ **(B) $(C - A)'$**

Ans. Given that $U = \{1, 2, \ldots 10\}$, $A = \{1, 2, 3, 5\}$, $B = \{2,4,6,7\}$ and $C = \{2, 3, 4, 8\}$

(A) $B \cup C = \{2,4,6,7\} \cup \{2, 3, 4, 8\} = \{2, 3, 4, 6, 7, 8\}$

Hence, $(B \cup C)' = \{1, 5, 9, 10\}$

(B) $C - A = \{2, 3, 4, 8\} - \{1, 2, 3, 5\} = \{4, 8\}$.

Hence, $(C - A)' = \{1, 2, 3, 5, 6, 7, 9, 10\}$

30. **If R is the set of all real numbers and Q is the set of all rational numbers, then what is $R - Q$?**

Ans. Given that R is the set of all real numbers and Q is the set of all rational numbers. Then, $R - Q = \{x : x \in R \text{ and } x \notin Q\}$ = set of all irrational numbers.

31. **If $a \in N$ such that $aN = \{ax : x \in N\}$. Describe the set $2N \cap 4N$.**

Ans. We have, $aN = \{ax : x \in N\}$

$\therefore \quad 2N = \{2x : x \in N\} = \{2, 4, 6, 8, \ldots\}$

and, $\quad 4N = \{4x : x \in N\} = \{4, 8, 12, 16, \ldots\}$

Hence, $2N \cap 4N = \{4, 8, 12, \ldots\}$

$= \{4x : x \in N\} = 4N$

32. **If $A = \{2, 4, 6, 8, 10 \ldots\}$, $B = \{1, 3, 5, 7, \ldots\}$ and N is the universal set, then find $A' \cup ((A \cup B) \cap B')$.**

Ans. Clearly, $(A \cup B) \cap B' = A$

[$\because$ A, B are disjoint sets]

$\therefore \quad A' \cup ((A \cup B) \cap B') = A' \cup A = N$

[the Universal set]

33. **Fill in the blanks.**

(A) $A \cup A' = \ldots$ **(B)** $\phi' \cap A \ldots$

(C) $A \cap A' \ldots$ **(D)** $U' \cap A = \ldots$

Ans. (A) $A \cup A' = U$ (B) $\phi' \cap A = A$

(C) $A \cap A' = \phi$ (D) $U' \cap A = \phi$

34. **Which of the following are examples of the empty set?**

(A) set of all even natural numbers divisible by 5.

(B) Set of all even prime numbers.

(C) $\{x : x^2 - 2 = 0$ and x is rational$\}$.

(D) $\{x : x$ is a natural number, $x < 8$ and simultaneously $x > 12\}$

(E) $\{x : x$ is a point common to any two parallel lines$\}$.

Ans. (C), (D) and (E) are examples of empty set.

(A) 10, 20, 30... are the elements that are even and also divisible by 5. So, it is not an empty set.

(B) 2 is the even prime number. So, it is not the empty set.

(C) $x^2 - 2 = 0$,

$\Rightarrow x^2 = 2$

$\Rightarrow x = \sqrt{2}, \sqrt{2}$ is an irrational number, hence it is an empty set.

(D) There is no number that is greater than 12 and less than 8, hence it is an empty set.

(E) There is no common point between two parallel lines, hence it is an empty set.

35. **Write all the elements of each of the following sets:**

(A) Set of prime number ≤ 11.

(B) $\{x : x \in Z$ and $x > -3$.

Ans. (A) Set of prime number $\leq 11 = \{2, 3, 5, 7, 11\}$

(B) $\{x : x \in Z \text{ and } x > -3\} = \{-2, -1, 0, 1, 2, \ldots\}$

36. **List all the proper subsets of the set $\{a, b, c\}$**

Ans. Given set is $\{a, b, c\}$.

The proper subsets of the given set are:

$\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, c\}$.

37. **If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$.**

Find $A \cap B$ and $A \cup B$.

Ans. We have, $A \cap B = \{3, 4\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

38. **Let $A = \{a, s, d, f\}$, $B = \{a, b, c, d, e\}$ and $U = \{a, b, c, d, e, f, s, t\}$.**

(A) $(A \cup B)'$ **(B)** $A' \cup B'$ **[Diksha]**

Ans. Given:

$A = \{a, s, d, f\}$, $B = \{a, b, c, d, e\}$ and $U = \{a, b, c, d, e, f, s, t\}$ $A \cup B = \{a, b, c, d, e, f, s\}$

(A) $(A \cup B)' = U - (A \cup B)$

$= \{a, b, c, d, e, f, s, t\} - \{a, b, c, d, e, f, s\} = \{t\}$

(B) $A' = U - A = \{b, c, e, s, t\}$

$B' = U - B = \{f, s, t\}$

$A' \cup B' = \{b, c, e, s, t, f\}$

39. **Write the set B = $\{x : x$ is a two digit numbers such, that the sum of its digits is 7$\}$.**

[Delhi Gov. QB 2021]

Ans. $B = \{16, 25, 34, 43, 52, 61, 70\}$.

40. **If $U = \{7, 8, 9, 10, 11, 12\}$, $A = \{8, 9\}$ and $B = \{9, 10, 11\}$. Find $A' \cup B'$.**

Ans. $A' = \{7, 10, 11, 12\}$

$B' = \{7, 8, 12\}$

$A' \cup B' = \{7, 8, 10, 11, 12\}$

41. **For any two sets A and B, prove that $A \cap (A' \cap B) = A \cap B$.**

Ans. $A \cap (A' \cap B) = (A \cap A') \cup (A \cap B)$

[By Distributive Law]

$= \phi \cup (A \cap B)$

$= A \cap B$.

Hence, proved.

42. **For any two sets A and B, prove that $A \cap (A \cup B)' = \phi$**

Ans. $A \cap (A \cup B)' = A \cap (A' \cap B')$

[By De Morgan's Law]

$= (A \cap A') \cap B'$

[By Associative Law]

$= \phi \cap B' = \phi$.

43. Write (–5, 9] in set-builder form. [Delhi Gov. QB 2021]

Ans. The initernal (–5, 9] can be written as $S = \{x : x \in R, -5 < x \le 9)$ in set builder form.

44. If $U = \{7, 8, 9, 10, 11, 12\}$, $A = \{8, 9\}$ and $B = \{9, 10, 11\}$. Find $A' \cup B'$.

Ans. $A' = U - A = \{7, 10, 11, 12\}$

$B' = U - B = \{7, 8, 12\}$

$A' \cup B' = \{7, 8, 10, 11, 12\}$

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

45. If $X = \{5, 6, 7, 8\}$, $Y = \{7, 8, 9, 10\}$, $Z = \{3, 4, 5, 6\}$. Find:

(A) $((X \cap Y) \cup Z)$

(B) $((X \cup Y) \cap Z)$

Ans. Given, $X = \{5, 6, 7, 8\}$, $Y = \{7, 8, 9, 10\}$, $Z = \{3, 4, 5, 6\}$

(A) $X \cap Y = \{5, 6, 7, 8\} \cap \{7, 8, 9, 10\} = \{7, 8\}$

$(X \cap Y) \cup Z = \{7, 8\} \cup \{3, 4, 5, 6\}$

$= \{3, 4, 5, 6, 7, 8\}$

(B) $X \cup Y = \{5, 6, 7, 8\} \cup \{7, 8, 9, 10\}$

$= \{5, 6, 7, 8, 9, 10\}$

$(X \cup Y) \cap Z = \{5, 6, 7, 8, 9, 10\} \cap \{3, 4, 5, 6\}$

$= \{5, 6\}$

46. Write the following in roster form.

(A) The set of all even prime numbers.

(B) $\{x : x \in N, x > 2$ and $x < 10\}$

(C) Set of all letters in the word MEERUT.

(D) $\{x : x$ is a natural number less than 6$\}$.

(E) $\{x : x$ is a positive integer and $x^2 < 40\}$.

Ans. (A) $\{2\}$

(B) $\{3, 4, 5, 6, 7, 8, 9\}$

(C) {M, E, R, U, T}

(D) $\{1, 2, 3, 4, 5\}$

(E) $\{1, 2, 3, 4, 5, 6\}$

47. Write the following sets in set-builder form:

(A) $\{5, 25, 125, 625\}$

(B) $\{2, 4, 6 \ldots\}$

(C) set of all even natural numbers

(D) set of all letters in the word 'TEACHER'.

Ans. (A) $\{x : x = 5^n, x \in N$ and $n \le 4$

(B) $\{x : x = 2n, n \in N\}$

(C) $\{x : x$ is an even natural number$\}$

(D) $\{x : x$ is a letter in word "TEACHER"$\}$

48. Given that $N = \{1, 2, 3, \ldots 100\}$, then

(A) Write the subset A of N, whose elements are odd numbers.

(B) Write the subset B of N, whose elements are represented by $x + 2$, where $x \in N$. [NCERT Exemplar]

Ans. Given, $N = \{1, 2, 3, \ldots, 100\} = \{x : x = n$ and $n \in N\}$

(A) $A = \{x \mid x \in N$ and x is odd$\} = \{1, 3, 5, 7, \ldots 99\}$

(B) $B = \{y \mid y = x + 2, x \in N\}$

The set whose elements are represented by $x + 2$ where $x \in N$ is obtained by putting $x = 1, 2, 3$ and so on in $y = x + 2$, we get

$y = x + 2 = 1 + 2 = 3$

$y = x + 2 = 2 + 2 = 4$

$y = x + 2 = 3 + 2 = 5$

$y = x + 2 = 4 + 2 = 6 \ldots$

$y = x + 2 = 100 + 2 = 102$

So, the required set will be $A = \{3, 4, 5, 6, \ldots 102\}$.

49. For any two sets A and B, prove that:

(A) $A \cup (B - A) = A \cup B$

(B) $(A \cap B) \cup (A - B) = A$

Ans. (A) $A \cup (B - A) = A \cup (B \cap A')$

$= (A \cup B) \cap (A \cup A')$

[By Distributive Law]

$= (A \cup B) \cap U = A \cup B.$

(B) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$

$= A \cap (B \cup B')$

[By Distributive Law]

$= A \cap U = A.$

50. Given $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6\}$ and $C = \{3, 5, 7\}$. Find $A - (B \cup C)$ and $A - (B \cap C)$.

Ans. Given

$A = \{1, 2, 3, 4, 5\}, B = \{2, 4, 6\}$ and $C = \{3, 5, 7\}$.

Then,

$B \cup C = \{2, 4, 6\} \cup \{3, 5, 7\} = \{2, 3, 4, 5, 6, 7\}$

$B \cap C = \{2, 4, 6\} \cap \{3, 5, 7\} = \phi$

Hence,

$A - (B \cup C) = \{1, 2, 3, 4, 5\} - \{2, 3, 4, 5, 6, 7\}$

$= \{1, 6, 7\}.$

And

$A - (B \cap C) = \{1, 2, 3, 4, 5\} - \phi = \{1, 2, 3, 4, 5\}$

51. Show that if $A \cup B = A \cap B$ it implies that $A = B$.

Ans. Given that, $A \cup B = A \cap B$ ...(i)

To show $A = B$.

Let $a \in A$ be any element.

$\Rightarrow \quad a \in A \cup B \quad [\because A \subseteq A \cup B]$

$\Rightarrow \quad a \in A \cap B \quad$ [Using (i)]

$\Rightarrow \quad a \in B$

Then, $\quad A \subseteq B \quad$ ...(ii)

Let $a \in B$ be any element

$\Rightarrow \quad a \in A \cup B \quad [\because B \subseteq A \cup B]$

$\Rightarrow \quad a \in A \cap B \quad$ [Using (i)]

$\Rightarrow \quad a \in A$

Thus, $\quad B \subseteq A \quad$ ...(iii)

From (ii) and (iii), we get

$A = B.$

52. Given that $E = \{2, 4, 6, 8, 10\}$. If n represents any member of E, then write the sets containing all numbers represented by the following:

(A) $2n + 2$ (B) n^3

Ans. (A) Here, $E = \{2, 4, 6, 8, 10\}$

A set containing elements in the form $2n + 2$:

If $n = 2 \Rightarrow 2n + 2 = 2 \times 2 + 2 = 6$

If $n = 4 \Rightarrow 2n + 2 = 2 \times 4 + 2 = 10$

If $n = 6 \Rightarrow 2n + 2 = 2 \times 6 + 2 = 14$

If $n = 8 \Rightarrow 2n + 2 = 2 \times 8 + 2 = 18$

If $n = 10 \Rightarrow 2n + 2 = 2 \times 10 + 2 = 22$

So, the required set is $\{6, 10, 14, 18, 22\}$

(B) A set containing elements in the form of n^3 are

If $n = 2 \Rightarrow n^3 = 2^3 = 8$

If $n = 4 \Rightarrow n^3 = 4^3 = 64$

If $n = 6 \Rightarrow n^3 = 6^3 = 216$

If $n = 8 \Rightarrow n^3 = 8^3 = 512$

If $n = 10 \Rightarrow n^3 = 10^3 = 1000$

So, the required set is $\{8, 64, 216, 512, 1000\}$.

53. If $P(A) = P(B)$, show that $A = B$.

Ans. Given, $P(A) = P(B)\ \forall a \in A$

$\Rightarrow \quad \{a\} \subset A$

$\Rightarrow \quad \{a\} \in P(A)$

$\Rightarrow \quad \{a\} \in P(B) \quad [\because P(A) = P(B)]$

$\Rightarrow \quad \{a\} \in B$

$\Rightarrow \quad \{a\} \subset B$

$\Rightarrow \quad A \subset B$

For all $b \in B$

$\Rightarrow \quad \{b\} \subset B$

$\Rightarrow \quad \{b\} \in P(B) \quad [\because P(A) = P(B)]$

$\Rightarrow \quad \{b\} \in P(A)$

$\Rightarrow \quad \{b\} \in A$

$\Rightarrow \quad \{b\} \subset A$

$\Rightarrow \quad B \subset A$

Since, $A \subset B$ and $B \subset A$

$\Rightarrow \quad A = B$

54. Are sets $A = \{1, 2, 3, 4\}$, $B = \{x : x = N \text{ and } 5 \le x \le 7\}$ disjoint? Justify? [Delhi Gov. QB 2021]

Ans. Given, $\quad A = \{1, 2, 3, 4\}$

And, $\quad B = \{x : x = N \text{ and } 5 \le x \le 7\}$

$= \{5, 6, 7\}$

Now, $\quad A \cap B = \{1, 2, 3, 4\} \cap \{5, 6, 7\} = \phi$

Hence, sets A and B are disjoint, because $A \cap B = \phi$.

55. In a class of 60 students, 25 students play cricket and 20 students play tennis, and 10 students play both the games. Find the number of students who play neither. [NCERT Exemplar]

Ans. Let U denote a set of all students in the class.

A denotes the set of students who play cricket and B denotes the set of students who play tennis.

We have,as per the question,

$n(U) = 60, n(A) = 25, n(B) = 20,$

$n(A \cap B) = 10$

Now,

$n(A' \cap B') = n[(A \cap B)'] = n(U) - n(A \cup B)$

$= n(U) - [n(A) + n(B) - n(A \cap B)]$

$= n(U) - n(A) - n(B) + n(A \cap B)$

$= 60 - 25 - 20 + 10 = 25.$

Hence, the number of students who play neither cricket nor tennis is 25.

56. In a class of 45 students, 25 students like Hindi, 20 like science and 11 like both. Find the number of students who like:

(A) either Hindi or science

(B) neither Hindi nor science.

Ans. Let U = all the students of the class,

H = students who like Hindi

S = Students who like Science

(A) The number of students who like either Hindi or science $= n(H \cup S)$

We have, $n(H \cup S) = n(H) + n(S) - n(H \cap S)$

$= 25 + 20 - 11$

$= 34$

(B) $\quad n(H' \cap S') = n(H \cup S)'$

$= U - n(H \cup S)$

$= 45 - 34$

$= 11$

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

57. **Let $A = \{1, 2, 3, 4\}$ $B = \{1, 2, 3\}$ and $C = \{2, 4\}$. Find all sets of X satisfying each pair of conditions:**

(A) $X \subset B$ and $X \not\subset C$

(B) $X \subset B$, $X \neq B$ and $X \not\subset C$

(C) $X \subset A$, $X \subset B$ and $X \subset C$

Ans. (A) We have,

$X \subset B$ and $X \not\subset C$

$\Rightarrow$ X is a subset of B but X is not a subset of C.

$\Rightarrow$ $X \in P(B)$ but $X \notin P(C)$

$\Rightarrow$ $X = \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$.

(B) We have,

$X \subset B$, $X \neq B$ and $X \not\subset C$

$\Rightarrow$ X is a subset of B other than B itself, and X is not a subset of C.

$X \in P(B)$

$\Rightarrow$ $X \in P(B)$, $X \notin P(C)$ but $X \neq B$

$\Rightarrow$ $X = \{1\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}$

(C) We have,

$X \subset A$, $X \subset B$ and $X \subset C$

$\Rightarrow$ $X \in P(A)$, $X \in P(B)$ and $X \in P(C)$

$\Rightarrow$ X is a subset of A, B, and C.

$\Rightarrow$ $X = \phi, \{2\}$.

58. **In a town of 10000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspapers. Find**

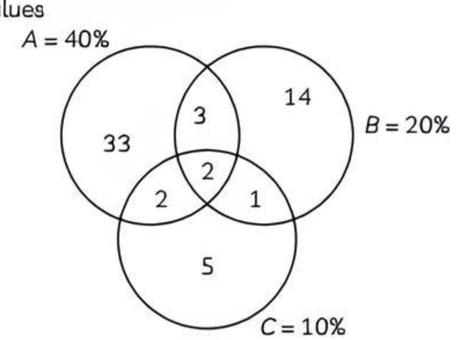
(A) the number of families which buy newspaper A only.

(B) the number of families which buy none of A, B and C. **[Diksha]**

Ans. Here,

$n(A) = 40\%$ $n(B) = 20\%$, $n(C) = 10\%$, $n(A \cap B) = 5\%$, $n(B \cap C) = 3\%$, $n(C \cap A) = 4\%$, $n[A \cap B \cap C] = 2\%$

We can draw a Venn-diagram using the above values



(A) $n(A \text{ only})$

$= n(A) - [n(A \cap B) + n(A \cap C)] + n(A \cap B \cap C)$

$= 40\% - [5\% + 4\%] + 2\%$

$= 40\% - [9\%] + 2\%$

$= 33\%$

Thus, the number of families who buy newspaper A only

$= 33\%$ of 10000

$= 3300$

(B) The number of families which buy none of is total families - those who buy either A, B or C

$= 100\% - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

$= 100\% - [40\% + 20\% + 10\% - 5\% - 3\% - 4\% + 2\%]$

$= 100\% - [60\%] = 40\%$

$\therefore$ Number of families, which buy none of A, B and C newspapers out of 10000 families are

$$= 10000 \times \frac{40}{100}$$

$= 4000$ families

59. **There are 230 students. 80 play football, 42 play soccer and 12 play rugby. 32 play exactly 2 sports and 4 play all three. How many students play none?** **[Diksha]**

Ans. We will calculate the number of students who play none of the sports by the formula which is given below:

Total students = students play football + students play soccer + students play rugby − (students who play exactly 2 sports) − 2 × (students who play all three sports) + students who play none

Putting the values in the above formula, we get,

$230 = 80 + 42 + 12 - 32 - 2 \times 4 +$ students who play none

Students who play none $= 230 - 80 - 42 - 12 + 32 + 8$

Students who play none = 136

Hence, the number of students who play none of the sport is 136.

60. **Prove that if $P \cup Q = R$ and $P \cap Q = \phi$ then $P = R - Q$.**

Ans. We have,

RHS $= R - Q$

$= (P \cup Q) - Q$

$= (P \cup Q) \cap Q'$
$= Q' \cap (P \cup Q)$
$= (Q' \cap P) \cup (Q' \cap Q)$
$= (Q' \cap P) \cup \phi$
$= Q' \cap P$
$= P \cap Q'$
$= P - Q$
$= P = \text{LHS}$ [as $P \cap Q = \phi$]

Hence, proved.

61. **Let P and Q be sets, if $P \cap X = Q \cap X = \phi$ and $P \cup X = Q \cup X$ for some set X. Show that $P = Q$.**

Ans. Given that, $P \cap X = Q \cap X = \varnothing$ and $P \cup X = Q \cup X$

To prove, $P = Q$

Using, $P \cup X = Q \cup X$ (given)

$$P \cap (P \cup X) = P \cap (Q \cup X)$$

Using distributive property,

$$(P \cap P) \cup (P \cap X) = (P \cap Q) \cup (P \cap X)$$
$$P \cup \phi = (P \cap Q) \cup \phi$$
$$P = (P \cap Q) \quad ...(i)$$

Again using, $P \cup X = Q \cup X$

$$Q \cap (P \cup X) = Q \cap (Q \cup X)$$

Using distributive property,

$$(Q \cap P) \cup (Q \cap X) = (Q \cap Q) \cup (Q \cap X)$$
$$(Q \cap P) \cup \phi = Q \cup \phi$$
$$Q = (Q \cap P)$$
$$Q = (P \cap Q) \quad ...(ii)$$

From (i) and (ii), we get

$$P = Q$$

Hence, proved.

62. **If $A = \{3, 6, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ and $D = \{5, 10, 15, 20\}$. Find:**

(A) $A - B$

(B) $B - C$

(C) $B - D$ **[Delhi Gov. SQP 2022]**

Ans. Given, $A = \{3, 6, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$ and $D = \{5, 10, 15, 20\}$.

(A) Here, $A = \{3, 6, 12, 15, 18, 21\}$ and
$B = \{4, 8, 12, 16, 20\}$
$A - B = \{3, 6, 15, 18, 21\}$

(B) Here, $B = \{4, 8, 12, 16, 20\}$ and
$C = \{2, 4, 6, 8, 10, 12, 14, 16\}$
$B - C = \{20\}$

(C) Here, $B = \{4, 8, 12, 16, 20\}$ and
$D = \{5, 10, 15, 20\}$
$B - D = \{4, 8, 12, 16\}$

63. **If $A = \{1, 3, 5, 7, 11, 13, 15, 17\}$, $B = \{2, 4, 6, 8, ... 18\}$ And U is universal set then find $A' \cup [(A \cup B) \cap B']$** **[Delhi Gov. QB 2021]**

Ans. Given, $A = \{1, 3, 5, 7, 11, 13, 15, 17\}$, $B = \{2, 4, 6, 8, ... 18\}$ and U is the universal set.

$A' = \{2, 4, 6, 8, 9....16\}$

$A \cup B = \{1, 3, 5, 7, 11, 13, 15, 17\} \cup \{2, 4, 6, 8....18\}$
$= \{1, 2, 3, 4, 5....17, 18\}$

And, $B' = \{1, 3, 5, 7, 9....17\}$

$(A \cup B) \cap B' = \{1, 2, 3, 4, 5....17, 18\} \cap \{1, 3, 5, 7, 9....17\}$
$= \{1, 3, 5, 7, 9....17\}$

Now, $A' \cup [(A \cup B) \cap B'] = \{2, 4, 6, 8, 9....16\} \cap \{1, 3, 5, 7, 9....17\}$
$= \{1, 2, 3, 4, 5, 6....18\}$
$= U =$ Universal Set

LONG ANSWER Type Questions (LA)

[**4** & **5** marks]

64. **In a competition, a school awarded medals in different categories, 36 medals in dance, 12 medals in dramatics and 18 medals in music. If these medals went to a total of 45 students and only 4 students got medals in all the three categories, how many received medals in exactly two of the categories?** **[Diksha]**

Ans. Let A = set of persons who get medals in dance.
B = set of persons who got medals in dramatics.
C = set of persons who got medals in music.

Given,
$n(A) = 36, n(B) = 12, n(C) = 18,$
$n(A \cup B \cup C) = 45$ and $n(A \cap B \cap C) = 4$

We know that the number of elements belonging to exactly two of the three sets A, B, C

$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3n(A \cap B \cap C)$
$= n(A \cap B) + n(B \cap C) + n(A \cap C) - 3 \times 4 \quad ...(i)$

Also,

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Therefore,

$$n(A \cap B) + n(B \cap C) + n(A \cap C) = n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C)$$

From (i),

$$n(A) + n(B) + n(C) + n(A \cap B \cap C) - n(A \cup B \cup C) - 12$$

$= 36 + 12 + 18 + 4 - 45 - 12$

$= 70 - 57 = 13$

65. There are 2000 students in a school. Out of these, 1000 play cricket, 600 play basketball and 550 play football, 120 play cricket and basketball, 80 play basketball and football and 150 play cricket and football and 45 play all three games. How many students play none of the games? [NCERT Exemplar]

Ans. Let U denotes a set of all students in the school.

A denotes the set of students who play cricket, B denotes the set of students who play basketball and C denotes the set of students who play football.

We have, $n(U) = 2000$, $n(A) = 1000$, $n(B) = 600$, $n(C) = 550$, $n(A \cap B) = 120$, $n(A \cap C) = 150$, $n(B \cup C) = 80$, $n(A \cap B \cap C) = 45$

Now, we know that

$$n(A' \cap B' \cap C') = n(U) - n(A \cup B \cup C)$$

$$= n(U) - [n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)]$$

$$= 2000 - [1000 + 600 + 500 - 120 - 150 - 80 + 45]$$

$$= 205$$

Hence, the number of students who play none of the games is 205.

66. In a group, 150 students know Hindi and 60 know English and 10 know both Hindi and English. If there are 30 students who know neither of the two languages, how many students are there in the group?

Ans. Let U denote a set of all students in the group.

A denotes the set of students who know Hindi and B denotes the set of students who know English.

We have, $n(A) = 150$, $n(B) = 60$, $n(A \cap B) = 10$,

$n(A' \cap B') = 30$.

Now, $n(A' \cap B') = n[(A' \cap B)']$

$= n(U) - n(A \cup B)$

$= n(U) - [n(A) + n(B) - n(A \cap B)]$

$= n(U) - n(A) - n(B) + n(A \cap B)$

$\therefore \quad n(U) = n(A) + n(B) - n(A \cap B) + n(A' \cap B')$

$= 150 + 60 - 10 + 30 = 230$

Hence, the number of students in the group is 230.

67. In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

Ans. Let M be the set of students who study mathematics, P be the set of students who study physics and C be the set of students who study chemistry.

Then,

$n(U) = 200$, $n(M) = 120$, $n(P) = 90$, $n(C) = 70$,

$n(M \cap P) = 40$, $n(P \cap C) = 30$, $n(C \cap M) = 50$

Also, $n(U) - n(M \cup P \cup C) = 20$

Or $n(M \cup P \cup C) = 200 - 20 = 180$

$$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(P \cap C) - n(C \cap M) + n(M \cap P \cap C)$$

$\Rightarrow \quad 180 = 120 + 90 + 70 - 40 - 30 - 50 + n(M \cap P \cap C)$

$\Rightarrow \quad 180 = 160 + n(M \cap P \cap C)$

$\Rightarrow \quad n(M \cap P \cap C) = 180 - 160 = 20$

So, the number of students who study all the three subjects is 20.

68. Let A, B and C be sets. Then show that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [NCERT Exemplar]

Ans. We first show that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$

Let $x \in A \cup (B \cap C)$.

Then $x \in A$ or $x \in B \cap C$

$\Rightarrow \quad x \in A$ or ($x \in B$ and $x \in C$)

$\Rightarrow \quad$ ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$\Rightarrow \quad$ ($x \in A \cup B$) and ($x \in A \cup C$)

$\Rightarrow \quad x \in (A \cup B) \cap (A \cup C)$

Thus, $\quad A \cup (B \cup C) \subset (A \cup B) \cap (A \cup C)$ ...(i)

Now we will show that

$$(A \cup B) \cap (A \cup C) \subset (A \cup C)$$

Let $x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow \quad x \in A \cup B$ and $x \in A \cup C$

$\Rightarrow \quad$ ($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$\Rightarrow \quad x \in A$ or ($x \in B$ and $x \in C$)

$\Rightarrow \quad x \in A$ or ($x \in B \cap C$)

$\Rightarrow \quad x \in A \cup (B \cap C)$

Thus, $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$ ...(ii)

So, from (i) and (ii)

We have $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$

Hence, proved.

69. **Let P be the set of prime numbers and let $S = \{t| 2^x - 1$ is a prime$\}$. Prove that $S \subset P$.**

Ans. Now the equivalent contrapositive statement of $x \in S \Rightarrow x \in P$ is $x \notin P \Rightarrow x \notin S$.

Now, we will prove the above contrapositive statement by contradiction method

Let $x \notin P \Rightarrow x$ is a composite number

Let us now assume that $x \in S$

$\Rightarrow \quad 2^x - 1 = m$ (where m is a prime number)

$\Rightarrow \quad 2^x = m + 1$

Which is not true for all composite numbers, say for $x = 4$ because $2^4 = 16$ which cannot be equal to the sum of any prime number m and 1.

Thus, we arrive at a contradiction $\Rightarrow x \notin S$.

Thus, when $x \notin P$, we arrive at $x \notin S$

So, $S \subset P$

Hence, proved.

70. **Using properties of sets and their complements prove that**

[Delhi Gov. SQP 2022]

(A) $(A \cup B) \cap (A \cup B') = A$

(B) $A - (A \cap B) = A - B$

(C) $(A \cup B) - C = (A - C) \cup (B - C)$

(D) $A - (B \cup C) = (A - B) \cap (A - C)$

(E) $A \cap (B - C) = (A \cap B) - (A \cap C)$

Ans. (A) $(A \cup B) \cap (A \cup B') = A$

LHS $= (A \cup B) \cap (A \cup B')$

$= A \cup (B \cap B')$ [By distributive law]

$= A \cup \phi$ $[\because B \cap B' = \phi]$

$= A =$ RHS

(B) $A - (A \cap B) = A - B$

LHS$= A - (A \cap B)$

$= A \cap (A \cup B)'$ $[\because A - B = (A \cap B)]$

$= A \cap (A' \cap B')$ [By Demorgan's law]

$= (A \cap A)' \cup (A \cap B')$

[By distributive law]

$= \phi \cup A \cap B'$ $[\because A \cap A' = \phi]$

$= A \cap B'$

$= A - B$

$=$ RHS

(C) $(A \cup B) - C = (A - C) \cup (B - C)$

Let $x \in (A \cup B) - C$

$x \in A \cup B$ and $x \notin C$

$x \in A$ or $x \in B$ and $x \notin C$

$x \in A - C$ or $x \in B - C$

$x \in (A - C) \cup (B - C)$

$(A \cup B) - C \subseteq (A - C) \cup (B - C)$ ...(i)

Now, let $x \in (A - C) \cup (B - C)$

$x \in (A - C)$ or $x \in (B - C)$

$x \in A$ and $x \notin C$ or $x \in B$ and $x \notin C$

$x \in (A \cup B)$ and $x \notin C$

$x \in (A \cup B) - C$

$(A - C) \cup (B - C) \subseteq (A \cup B) - C$ ...(ii)

From eq. (i) and (ii)

$(A \cup B) - C = (A - C) \cup (B - C)$

(D) $A - (B \cup C) = (A - B) \cap (A - C)$

Let $x \in A - (B \cup C)$

$x \in A$ and $x \notin (B \cup C)$

$x \in A$ and $x \notin B$ or $x \notin C$

$x \in A - B$ and $x \in A - C$

$x \in (A - B) \cap (A - C)$

$A - (B \cup C) \subseteq (A - B) \cap (A - C)$ ...(i)

Now, let $x \in (A - B) \cap (A - C)$

$x \in A - B$ and $x \in A - C$

$x \in A$ and $x \notin B$ and $x \in A$ and $x \notin C$

$x \in A$ and $x \notin (B \cup C)$

$x \in A - (B \cup C)$

$\Rightarrow (A - B) \cap (A - C) \subseteq A - (B \cup C)$ ...(ii)

From eq. (i) and (ii)

$A - (B \cup C) = (A - B) \cap (A - C)$

(E) $A \cap (B - C) = (A \cap B) - (A \cap C)$

Let $x \in A \cap (B - C)$

$x \in A$ and $x \in B - C$

$x \in A$ and $x \in B$ and $x \notin C$

$x \in (A \cap B)$ and $x \notin (A \cap C)$

$A \cap (B - C) \subset (A \cap B) - (A \cap C)$ ...(i)

Now, let $x \in (A \cap B) - (A \cap C)$

$x \in (A \cap B)$ and $x \notin (A \cap C)$

$x \in A$ and $x \in B$ and $x \notin A$ and $x \notin C$

$x \in A$ and $x \in (B - C)$

$x \in A \cap (B - C)$

$\Rightarrow \quad (A \cap C) - (A \cap C) \subset A \cap (B - C)$ ...(ii)

From eq. (i) and (ii)

$A \cap (B - C) = (A \cap B) - (A \cap C)$